CMIS 2016

CONTACT MECHANICS INTERNATIONAL SYMPOSIUM

Book of Abstracts

May 11-13, 2016
Warsaw, Poland

8th
Contact Mechanics International Symposium (CMIS 2016)

May 11-13, 2016
Warsaw, Poland

Book of Abstracts

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WEDNESDAY 11.05.2016

8:00–9:00 REGISTRATION

9:00–9:40 CONFERENCE OPENING

9:40–10:40 **WED 01** SESSION

9:40–10:00	Y. Renard *Some fictitious domain approaches for the contact of deformable structures* (P005)
10:00–10:20	P. Massin, G Ferté and N. Moës *Modelling strong and weak discontinuities with X-FEM* (P041)
10:20–10:40	M. Hiermeier, W.A. Wall and A. Popp *A robust augmented Lagrangian mortar-type formulation for finite deformation contact problems* (P012)

10:40–11:20 COFFEE BREAK

11:20–12:40 **WED 02** SESSION

11:20–11:40	P. Wriggers , W. Rust and B.D. Reddy *A Virtual Element Method for Contact* (P009)
11:40–12:00	R. Kruse, N. Nguyen-Thanh, L. De Lorenzis, P. Wriggers *Isogeometric frictionless contact analysis with the third medium method* (P023)
12:00–12:20	R. Mlika, Y. Renard and F. Chouly *An unbiased Nitsche's approximation of contact between two elastic structures* (P011)
12:20–12:40	Z. Dostal, T. Kozubek, T. Brzobohaty, V. Hapla, D. Horak, A. Markopoulos, O. Vlach *Massively parallel algorithms for contact problems - theory and experiments* (P010)

12:40–14:20 LUNCH BREAK

14:20–15:40 **WED 03** SESSION

14:20–14:40	S. Stühler and P. Eberhard *Simulation of complex-shaped particle geometries with the discrete element method* (P027)
14:40–15:00	J. Rojek, K. Jurczak, S. Nosewicz, D. Lumelskyj and M. Chmielewski *Contact models for discrete element simulation of the initial powder compaction in a hot pressing process* (P038)
15:00–15:20	D. Horak, Z. Dostal, V. Hapla, O. Vlach, L. Pospisil, M. Cermak, R. Sojka *PERMON's scalable quadratic programming solvers for contact problems based on TFETI* (P029)
15:20–15:40	R. Vodicka, V. Mantic and T. Roubicek *A new procedure for the solution of quasistatic frictional contact problems by the symmetric Galerkin boundary element method and quadratic programming* (P043)

WEDNESDAY 11.05.2016

15:40–16:20 COFFEE BREAK

16:20–18:00 **WED 04** SESSION

16:20–16:40	M. Cocou *A variational analysis of a dynamic contact problem with Coulomb friction* (P032)
16:40–17:00	S. Migórski *History-dependent variational-hemivariational inequalities with applications to dynamic contact problem in viscoelasticity* (P002)
17:00–17:20	P. Krejc, A. Petrov *Existence and uniqueness results for dynamic thermo-elsato-plastic contact problems* (P016)
17:20–17:40	A. Rodriguez Aros *Models of elastic shells in contact with a rigid foundation* (P021)
17:40–18:00	A. Ochal and S. Migórski *Dynamic frictional contact of a thermoviscoelastic body with an obstacle* (P015)

18:20–20:20 GET-TOGETHER PARTY

THURSDAY 12.05.2016

	9:00–10:40 **THU 01** SESSION
9:00–9:20	M.L. Raffa, F. Lebon and G. Vairo *Interfacial stiffnesses of rough surfaces in no-sliding contact via an imperfect interface approach* (P007)
9:20–9:40	R. Buczkowski, M. Kleiber and A. Rzeczycki *A finite element study of the surface roughness in elasto-plastic shrink fitted joint* (P045)
9:40–10:00	I. Argatov, R. Pohrt, and V.L. Popov *Multi-scale asymptotic modelling of a cluster of adhesive micro-contacts* (P001)
10:00–10:20	N. Papadogianni, N. Kaminakis, G.A. Drosopoulos and G.E. Stavroulakis *Contact assisted auxetic microstructures* (P028)
10:20–10:40	S. Kucharski, G. Starzynski *Flattening of loaded rough surfaces: normal contact versus sliding contact* (P046)

10:40–11:20 COFFEE BREAK

	11:20–13:00 **THU 02** SESSION
11:20–11:40	U. Nackenhorst, R.L. Gates, M. Bittens, and R. Beyer *On the homogenization of rubber-rough-surface-contact using an adaptive stochastic collocation scheme* (P020)
11:40–12:00	P. Wagner, P. Wriggers, H. Clasen and C. Prange *Validation of a multiscale FEM approach for elastomer friction on rough surfaces* (P024)
12:00–12:20	K. Houanoh, H.-P. Yin and Q.-C. He *Contact of viscoelastic bodies with periodically wavy surfaces* (P040)
12:20–12:40	I. Goryacheva, E. Torskaya *Contact of multi-level periodic system of indenters with homogeneous and two-layered elastic half-space* (P031)
12:40–13:00	A. Chudzikiewicz, A. Myśliński *Elasto-plastic rolling contact problems with nonhomogeneous materials* (P022)

13:00–14:40 LUNCH BREAK

	14:40–16:00 **THU 03** SESSION
14:40–15:00	Z. Mróz, S. Kucharski *Anisotropic friction and wear rules with account for anisotropy evolution* (P044)
15:00–15:20	C. Stolz *Integral equation with variations of domain: applications to wear contact* (P036)
15:20–15:40	P. Farah, W.A. Wall and A. Popp *Towards an implicit finite wear formulation for non-smooth contact geometries* (P013)
15:40–16:00	A. Zmitrowicz *Modelling of wear of materials in terms of variational methods* (P019)

16:00–16:40 COFFEE BREAK

16:40–19:40 EXCURSION

FRIDAY 13.05.2016

9:00–10:40	**FRI 01** SESSION
9:00–9:20	R. Vodicka, T. Roubicek and V. Mantic *Modelling of frictional cohesive contacts* (P008)
9:20–9:40	A. Papangelo, M. Ciavarella and J. Barber *On a Griffith condition for the stick-slip boundary* (P004)
9:40–10:00	N. Terfaya, M. Raous, A. Berga *A bipotential method coupling contact, friction and adhesion* (P006)
10:00–10:20	G. Del Piero *The two fundamental fracture modes of one-dimensional bars* (P014)
10:20–10:40	I.P. Shatskyi *Cracks closure in thin plates and shells: mixed problems and analytical solutions* (P030)

10:40–11:20 COFFEE BREAK

11:20–12:40	**FRI 02** SESSION
11:20–11:40	V.A. Yastrebov, D.S. Kammer *Elastodynamic frictional sliding of an elastic layer on a rigid flat: stick-slip pulses and opening waves* (P037)
11:40–12:00	D. Schurr and P. Eberhard *Contact calculation in elastic multibody gear systems using different kind of model order reduction methods* (P026)
12:00–12:20	L. Rodríguez-Tembleque, F.C. Buroni, A. Sáez and F.M.H. Aliabadi *Indentation response of magneto-electro-elastic materials under frictional contact* (P018)
12:20–12:40	J. Lengiewicz, M. Kursa, P. Hołobut *Two-domain contact model of volumetric actuators* (P047)

12:40–14:20 LUNCH BREAK

14:20–16:00	**FRI 03** SESSION
14:20–14:40	H. Ben Dhia, S. Du *Multi-numerics and multi-physics modeling of multiscale contact problems* (P042)
14:40–15:00	J. Gwinner and D. Natroshvili *Contact problems in piezoelectricity – Mathematical modelling and boundary element approximation* (P033)
15:00–15:20	A.B. Kunin, S. Loehnert, P. Wriggers *Thermo-mechanical contact between crack surfaces in the eXtended Finite Element Method* (P035)
15:20–15:40	R.A. Sauer *Contact algorithms for liquid droplets* (P039)
15:40–16:00	S. Stupkiewicz, J. Lengiewicz, P. Sadowski and I. Temizer *Finite deformation effects in soft elastohydrodynamic lubrication problems* (P048)

16:00–16:40	COFFEE BREAK
18:00	TRANSFER TO JABŁONNA PALACE
19:00–	CONFERENCE DINNER

Abstracts

Some fictitious domain approaches for the contact of deformable structures

Yves Renard

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Summary: Two stragegies for the approximation of contact problems of elastic structures will be described and compared: a stabilised Lagrange multiplier method and a Nitsche based method. Both theoretical and numerical aspects will be discussed.

Introduction

The aim of the presentation will be to compare two different strategies to take account of the unilateral contact condition between two elastic body surfaces in a fictitious domain framework (of the type introduced in [5]) with a cut finite element method (see Figure 1).

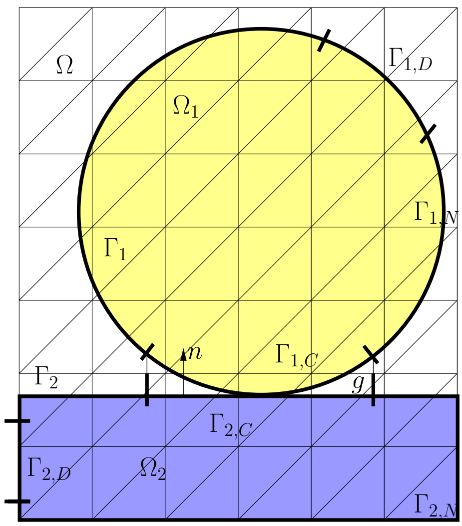

Figure 1: Exemple of fictitious domain approach: a single mesh for two elastic solids

This means in particular that the considered mesh will not be conformal to the two contact surfaces. Modeling situations where such methods are useful are quite common: contact of elastic bodies in fluid-structure interaction problems, shape optimization with contact, contact surfaces with very complex geometries, crack propagation, etc.

The first proposed strategy to approximate the contact condition will be a Lagrange multiplier method (see [1]). The difficulty with such a method is to satisfy a LBB condition that can be obtained either by carefully adjusting the discretization space for the multiplier, or by using a stabilization method. Some convergence results will be presented for the method with stabilization.

The second considered strategy will be a Nitsche-based method (see [2–4]). After the description of how this method is obtained in the case of unilateral contact and after discussing the need of a stabilization in this context, optimal *a priori* error estimate results will be presented.

References

[1] S. Amdouni, M. Moakher, Y. Renard. A stabilized Lagrange multiplier method for the enriched finite element approximation of Tresca contact problems of cracked elastic bodies. *Comp. Meth. Appl. Mech. Engng.*, 270:178-200, (2014).

[2] F. Chouly, P. Hild and Y. Renard, Symmetric and non-symmetric variants of Nitsches method for contact problems in elasticity: theory and numerical experiments. *Math. Comp*, 84:1089–1112 (2015).

[3] F. Chouly, M. Fabre, P. Hild, J. Pousin and Y. Renard. Residual-based a posteriori error estimation for contact problems approximated by nitsche's method. submitted.

[4] M. Fabre, J. Pousin, Y. Renard. A fictitious domain method for frictionless contact problems in elasticity using Nitsche's method. submitted.

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Modelling strong and weak discontinuities with x-fem

P. Massin[1], G Ferté[1,2] and N. Moës[2]

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Summary: modelling strong or weak discontinuities with x-fem involves specific treatment with respect to the satisfaction of the inf-sup condition. An augmented lagrangian approach is presented next. It has been applied to model bi-material interfaces, and interfaces with cohesive adhesion or contact-friction with small or large interface displacements.

Introduction

Ji and Dolbow were the first to introduce Lagrange multipliers in the X-FEM so as to enforce constraints on an interface [1]. They reported that a naive discretization triggers oscillations of the multipliers and a loss of convergence and so does high-stiffness penalization. The critical point lies in a discrete inf-sup condition, which determines the stability of the formulation. In X-FEM, this condition is even more technical to verify because of the non-conformity of the mesh and is violated for a naive discretization. There are several ways to restore it in the literature, namely,

(1) An enrichment of the displacement basis with bubble function on the interface in [2];

(2) The definition of a reduced Lagrange multiplier space, which was improved in a sequence of three papers by Moës and al [3], Géniaut and al [4] and Béchet and al [5]. Those spaces which were first designed for use with linear elements have been recently extended to quadratic elements in [6].

These are called stable formulations, in that they work out the discrete spaces directly, as opposed to stabilized formulations, which circumvent the inf-sup condition with a modification of the variational form by:

(1) An adaptation of Nitsche's approach to the X-FEM by Hansbo, further developed by [7]. In this approach, no Lagrange multiplier is introduced in the formulation: the normal flux directly plays its role and a stabilization term on the jump is added to avoid oscillations;

(2) The addition of a stabilization term on the multiplier/flux discrepancy into the weak formulation by [8] based on Barbosa and Hughes approach. A connection was made by Stenberg between this approach and Nitsche's one ;

(3) A three-field approach with ellipticity enhancing terms by Gravouil and al [9].

In the following, a Lagrange multiplier technique, convenient to access directly the pressure field on the interface is implemented

Lagrange multiplier with x-fem

Studying interface problems with X-FEM and Lagrange multipliers, up to the second order with curved interfaces has led to a threefold task,

(1) Designing both optimal and stable reduced multiplier space suite for both linear and quadratic displacements;

(2) Deriving theoretical a priori estimates for stable formulations of X-FEM with Lagrange multipliers, which has been done up to now solely for stabilized

formulations;

(3) Including the influence of the geometry representation in those estimates.

Interesting results were obtained [6, 10]. For bi-material interfaces with quadratic elements optimal convergence rates are obtained using the Lagrangian approach with gluing conditions while the introduction of a weak discontinuity in the enrichment of the x-fem formulation results suboptimal, which is not the case for linear elements. For strong discontinuous interfaces optimal convergence rates were obtained in all cases. Finally for large sliding, just the linear reduced multiplier was shown to give correct results. Current investigations try to address this issue.

Application to cohesive interface

The algorithms we developed were used recently to tackle crack propagation in 3D with an initially unknown crack path [11]. The cohesive model that is used allows initial perfect adherence. It relies on the use of the XFEM-suited reduced space of Lagrange multipliers previously mentioned, on the use of a mortar formulation to write the cohesive law from quantities defined over this space in an appropriate manner, and finally on a lumping strategy leading to block-wise diagonal operators. With all those ingredients it was then possible to carry out mixed mode simulations to reproduce 3D non planar crack paths in good accordance with experiments from the literature.

References

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[11] 3D crack propagation with cohesive elements in the extended finite element method, G. Ferté, P. Massin, N. Moës, *Comput. Methods Appl. Mech. Engrg.*, 300:347–374, 2016.

A robust augmented Lagrangian mortar-type formulation for finite deformation contact problems

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Summary: A globally convergent variation of the Newton method is presented, which is capable of dealing with large initial penetrations and improves the overall non-linear solver performance for highly non-linear contact problems. The global convergence property is based on an adapted filter line search method.

Contact formulation and motivation

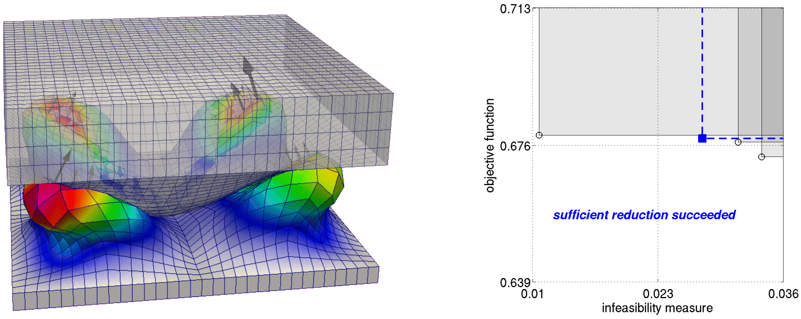
Contact problems are of huge interest in many engineering applications and at the same time they are one of the toughest problems to solve in solid and structural mechanics. The complexity mainly arises from the non-smooth multi-valued contact constraints. The discrete problem can be derived from a weak formulation based on an augmented structural contact potential followed by a variationally consistent finite element discretization, e.g. as provided by mortar methods [1, 2].

The contact constraints can be reformulated as a nonlinear C^1-continuous augmented saddle point problem [3]. This penalty duality formulation is highly favorable as long as the set of unknowns is sufficiently close to the solution point. However, there are still situations, where the non-linear solution process can stagnate or even diverge. These problems can not only be observed for very large load steps, but also for quite common, practically relevant step sizes. The crucial point is the Newton-Raphson convergence radius of the displacement and Lagrange multiplier unknowns [4]. Even under the consideration of globalization techniques, such as the line search routine, it can become troublesome to find a monotonically decreasing solution path with respect to the prescribed objectives.

Variation of the Newton method

In [5] a non-consistent start-up procedure for a pure displacement based penalty node-to-segment formulation was presented, which for the first time analyzes these problems in-depth. The authors identify terms of the linearization, which will improve the overall performance, if they are neglected during the critical pre-asymptotic phase.

In contrast to the penalty approach [5], a slightly different idea will be presented here, which is capable of dealing with the additional complexity arising from the exact enforcement of the constraints by Lagrange multipliers. In the pre-asymptotic phase the new algorithm uses the so-called first order steepest ascent method, since in general a first order iteration is more robust and reliable than a second order iteration. The presented variation of the Newton method [4] gives us the opportunity to define a natural transition between the global first order iteration and the local second order iteration schemes. All occurring terms can be easily interpreted. Furthermore, the system size can be reduced during the pre-asymptotic phase without the need of dual shape functions [1]. A suitable switching criterion will be presented.



(a) (b)

Figure 1: Sine-shaped globalization example: (a) deformed configuration of the two body contact problem during the non-linear iteration process, (b) visualization of the filter acceptance test.

Filter line search approach

The new approach is well-suited for a large variety of contact problems. But there are still problems, for which the non-linear solver can diverge, if no safe-guarding strategies are provided. For constrained problems the two distinct goals of minimization of the objective function and satisfaction of the constraints have to be considered. To account for these goals a so-called filter line search technique [6] will be presented, which is based on ideas of multi-objective optimization. In Figure 1, a typical successful iteration step of such a filter safe-guarded non-linear solution procedure is visualized. The deformed configuration and the corresponding acceptance check for the two opposing objectives are shown in the left and right graph, respectively.

References

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A virtual element method for contact

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Abstract: Many different approaches were developed over the last four decades to formulate contact constraints for numerical simulation methods. Especially in the finite element approach the method of Lagrange multipliers and the penalty method were implemented in most of the simulation packages that are available for the treatment of contact problems undergoing small and finite deformations. There is a large amount of work in which different formulations were pursued to design robust and efficient contact algorithms.
With the virtual element method (VEM) a new discretization was developed that leads in the linear case to an exceptional efficient and stable formulation for non-matching contact interfaces. In this paper we explore the possibilities opened up by the virtual element method for the discretization of contact interfaces. The beauty of the virtual element method is that different numbers of nodes can be used to define an element. This characteristic fits extremely well into a general contact formulation with non-matching meshes, since it allows to formulate a node-to-node contact approach. Hence a rather simple algorithm follows that transforms the contacting meshes in matching meshes where non-matching interfaces occur. With this the classical contact procedures for node-to-node normal contact based on linear shape functions can be used within the virtual element method.

Isogeometric frictionless contact analysis with the third medium method

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In this study, the recently proposed third medium method for frictionless contact is combined with isogeometric analysis. The role of the parameters of the material model is systematically investigated, with the aim of achieving best agreement with known contact pressure distributions, as well as nearly frictionless contact in large-scale sliding.

Introduction

A wide range of methods are available to deal with contact mechanics within a finite element discretization setting, for an overview see e.g. [1]. Despite the vast amount of research devoted to the topic, the non-smooth nature of the contact phenomenon still represents a challenge for the computational analyst. To overcome the associated problems, Wriggers et al. [2] recently proposed a new method in which the space between the bodies potentially in contact is filled with a third medium, described as a fictitious material featuring an isotropic /anisotropic behavior with changing directions and characteristics. By proper choice of the constitutive behavior of the third medium, interpenetration of the bodies is prevented, whereas the behavior prior to the onset of contact remains unaffected.

In this study, we pursue the following goals:

- systematically investigate the effect of the parameters contained in the material model of the third medium in frictionless contact without or with a significant amount of sliding, with the objective to find an optimal parameter combination for frictionless contact;
- explore the use of the third medium approach within the framework of isogeometric analysis (IGA) and the related advantages, as well as verify the possibility to achieve higher-order spatial convergence through the higher-order discretization of the continuum;
- investigate self contact as a attractive application of the third medium approach for porous geometries.

IGA, first proposed in [3], guarantees higher smoothness of the discretization and increased robustness under sever mesh distortion [4].

Parameter study

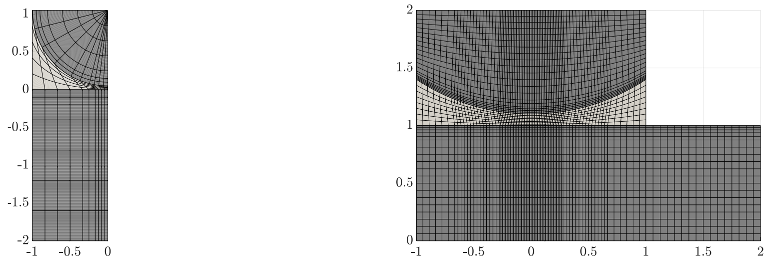
The effect of the numerous parameters of the third medium material is explored on two examples: Hertzian normal contact (Fig.1a) and large-scale sliding of an elastic cylinder (Fig.1b). For a large number of parameter combinations, the following measures are derived: 1) the difference in contact pressure relative to the analytical known solution; 2) the remaining gap size; 3) the computational effort expressed as total number of Newton-Raphson iterations needed; 4) the maximum amount of sliding before iterative convergence can no longer be achieved; 5) the apparent friction coefficient in sliding as ratio between tangential and normal reaction force. Multidimensional linear regression was utilized to obtain estimates of the relative effect of the material model on these parameters.

The results can be summarized as follows: The presence of the medium between the bodies in contact creates a smoothing effect on the pressure distribution, but good agreement with the analytical solution can still be achieved in the region of highest contact pressure, see Fig. 2a. Agreement is best if both the isotropic and the anisotropic stiffness are chosen as low as possible, but this choice, especially in the isotropic part, is limited by the increase in computational cost, at least under force-controlled loading as employed.

In sliding contact, similar observations can be made. A low isotropic (especially with respect to deviatoric deformation) and anisotropic stiffness leads to low apparent friction coefficients, but a low isotropic stiffness reduces the maximum amount of sliding possible.

The smooth nature of the solution found with the method in combination with the use of higher-order discretizations leads to the possibility to achieve higher-order spatial convergence rates. Finally, self contact appears to be an attractive application of the third medium approach for porous geometries.

In the present work such as in the original paper, a purely elastic behavior was assumed for the third medium, which was consistent with the study of frictionless contact. In order for frictional contact to be tackled with a third medium approach, a source of dissipation must be incorporated in the material modeling of the third medium, e.g. through an elasto-plastic constitutive behavior. This and additional aspects are open questions for further research.



(a) (b)

Figure 1: Exemplary geometries for evaluation of the third medium (shown in light grey) method in normal contact (a) and finite sliding (b).

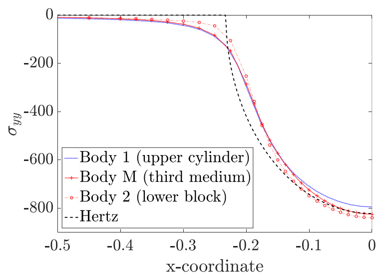


Figure 2: Comparison in contact pressure between third medium method and analytical solution.

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An unbiased Nitsche's approximation of contact between two elastic structures

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In this presentation we introduce a new unbiased formulation of the two elastic bodies contact problem and we present the analysis and numerical tests performed in the small strain framework. In the second part of the study, an extension of the method to the large strain context is presented and tested.

Introduction

Most of the numerical methods dedicated to the contact problem involving two elastic bodies are based on the master/slave paradigm. It results in important detection difficulties in the case of self-contact and multi-body contact, where it may be impractical, if not impossible, to a priori nominate a master surface and a slave one. In this work we introduce an unbiased method for the finite element approximation of frictional contact between two elastic bodies in the small deformation framework then extend it to large deformations context. The formulation is based on the Nitsche's method that is a promising method to treat unilateral contact. This method has the advantage of being consistent without using Lagrange multipliers. We carry out the numerical analysis of the method, and prove its well-posedness and optimal convergence in the H^1-norm.Numerical experiments are performed to assess the robustness and the performance of the method. Since Nitsche's method has not been used for large deformations, it is necessary to formulate and test it in this framework for self-contact and two bodies contact. To prove the accuracy of the method for large deformations, we test it with several classic numerical tests.

An unbiased Nitsche's approximation of the frictional contact in the small strain framework

We consider two bodies Ω^i expected to income into contact on the boundaries Γ_C^i. In order to obtain an unbiased formulation of the contact problem we prescribe the contact conditions deduced form the Signorini problem conditions (see [5]) and the Tresca friction conditions on the two surfaces Γ_C^i in a symmetric way and we will integrate on both of them. The derivation of a Nitsche-based method comes from a reformulation of the contact conditions (see for instance [2] and [3]). The use of Nitsche's method allows us to divide the contact effort equitably on both of contact surfaces using the second Newton law since Nitsche's method uses the contact stress as a multiplier. Similarly, a simple adaptation to Tresca's friction of the Nitsche-based finite element Method is proposed in [1]. All this results leads to an unbiased finite element formulation in which contact and friction are incorporated weakly on both contact boundaries Γ_C^i", through appropriate extra terms. To prove the efficiency of the method, we carry out some mathematical analysis on it. We prove, at first, the formal equivalence between strong and variational formulations for the continuous problem. Since the construction of the method consists in particular in the splitting of the contact terms into two parts, this step is necessary to ensure the coherence of the formulation. We prove also the consistency, the well-posedness and the optimal convergence of the method in H^1-norm. We provide,as well, numerical verification through several two/three-dimensional numerical tests. The tests cover a study of convergence with different values of the generalization parameter θ and the Nitsche's parameter γ_0. The open source environment GetFEM++[4] is used to perform the tests. All the details of the method's constraction and Analysis are given on the article [4]

Nitsche's method in large deformations framework

In the large deformation framework Nitsche's method has not been used for the contact problem; so we need to formulate and test this method for the biased and unbiased cases before using it in the self-contact problem. Using the same approach as for small deformations, we formulate, firstly, the frictionless problem with the non symmetric variant ($\theta = 0$). We test the accuarcy of the method through some classic numerical tests such as: the Talor Pach test, Hartizian contact, shallow ironing, elastic half ring and two crossed tubes. This tests are often used in the literature (see [7] and[6] and references therein). We illustrate the results of those test in the figure 1 where we present the deformed configuration of the Crossed tubes test. In this example we test the method for the two bodies contact as well as self-contact configuration.

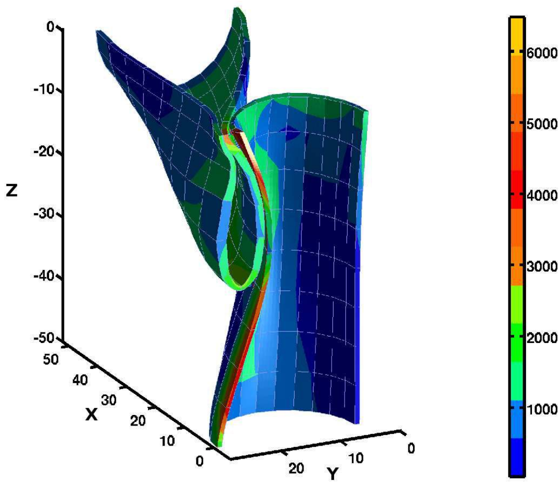

Figure 1: Contour plot of Van Mises stress on the deformed configuration of the crossed tubes test at 29 mm of relative displacement.

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Massively parallel algorithms for multibody contact problems – theory and experiments

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Summary: An overview of the recently developed TFETI based scalable massively parallel algorithms for the solution of contact problems is presented. The theory covers the frictionless problems, problems with a given (Tresca) friction, and transient problems of elasticity, as well as the sensitivity analysis and problems with plasticity. The efficiency of the algorithms is demonstrated by numerical experiments.

Introduction

We start with a short overview of the development od the TFETI/TBETI based scalable algorithms for the solution of contact problems and assessment of their capability to solve effectively extremely large and complex problems. Recall that TFETI differs from the classical FETI or FETI2 as introduced by Farhat and Roux by imposing the prescribed displacements by the Lagrange multipliers and treating all subdomains as "floating".The theory covers the frictionless problems, problems with a given (Tresca) friction, and transient problems of elasticity, as well as the sensitivity analysis and problems with plasticity. Let us recall that the algorithm is numerically scalable if the cost of the solution is nearly proportional to the number of unknowns, and that the algorithm enjoys numerical scalability if the time of the solution can be reduced nearly proportionally to the number of available processors.

Then we briefly overview our in a sense optimal algorithms for the solution of the resulting quadratic programming and QPQC (quadratic programming - quadratic constraints) problems. A unique feature of these algorithms is their capability to solve the class of such problems with homogeneous equality constraints and separable inequality constraints in $O(1)$ matrix–vector multiplications provided the spectrum of the Hessian of the cost function is in a given positive interval (see [1], [2], and [3]).

Finally we put together the above results to develop scalable algorithms for the solution of the above problems [4], [5], [6], [7], and [8]. A special attention is paid to the implementation details, such as stable and efficient evaluation of the action of the generalized inverse of the stiffness matrices of floating domains, re-normalization based scaling for jumping coefficients, variationally consistent discretization of contact conditions, preconditioning, massively parallel implementation, etc. We illustrate these results by massively parallel solution of academic benchmarks discretized by billions of nodal variables and by the solution of difficult real world problems, such as analysis the roller bearings in with 73 bodies under nonsymmetric loading or yielding clamp connection. We conclude by discussing the application of our scalable algorithms to the solution of other problems (Coulomb friction, contact shape optimization, contact with plasticity) and current research.

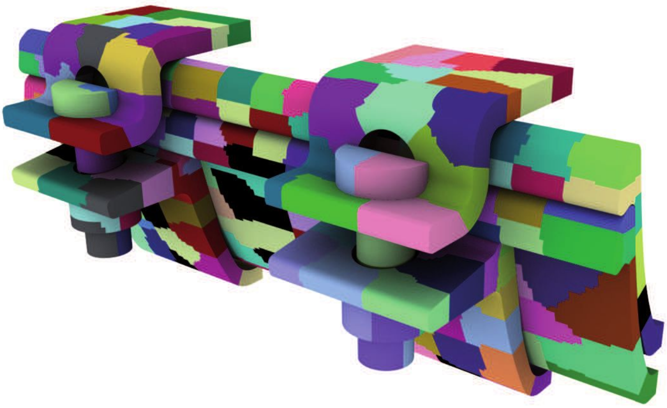

Figure 1: Domain decomposition of yielding clamp connection

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Simulation of complex-shaped particle geometries with the discrete element method

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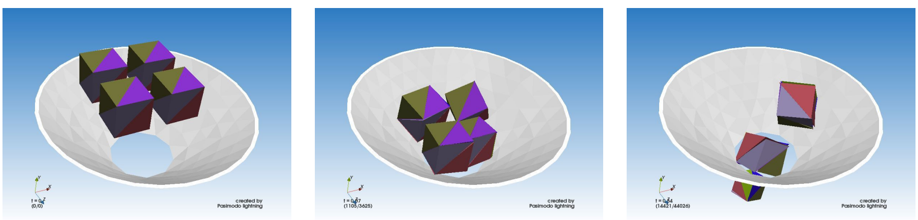
Summary: We present a particle based method to simulate complex particle geometries with features such as non-convex surfaces or holes. The approach is based on tetrahedral particles and includes a particle clustering mechanism. A suitable collision detection and contact force calculation algorithm is introduced and a description for considering deformations of the particles is given.

Numerical simulations of granular materials which are found in food industry process or in geotechnical science often cannot be handled properly by mesh-based simulation methods such as the finite element method or computational fluid dynamics. Modeling of crack growth or contact evaluation requires extensive remeshing or fine discretization of the geometry. To tackle these problems in an efficient way the meshfree Discrete Element Method (DEM) is commonly used [1]. DEM models simulate the problems with discrete particles that may interact with other particles and may be subjected to body forces. Most of the DEM codes use rigid particles that are spherical in shape. The description of the geometry and the interaction between these spherical particles is relatively straightforward and the respective numerical implementations are very efficient. However, the simplification by using perfect spherical particles involves drawbacks. The geometrical representation lacks certain important features such as edges, surfaces, holes. Therefore, advanced approaches are applied. Building particle clusters of spherical particles still benefiting of the aforementioned numerical advantages of spherical particles [2, 3] is one possibility. A better geometrical representation is achieved with descriptions based on sphero-polygons [4], super-ellipsoids [5] or inequalities [6] in conjunction with worse numerical efficiency. Although these approaches imply a better geometrical representation, the particles are treated as rigid and deformations are neglected.



(a) (b) (c)

Figure 1: Flow of coupled deformable tetrahedral particles through a hopper: (a) initial state, (b) bridging effect slowing down the particle movement, (c) full discharge.

Within this contribution, the discrete particles are described with tetrahedra - the simplest convex polyhedron. One advantage of the geometrical description with tetrahedral particles is their ability to represent the desired domain by particle clusters. Additional, standard mesh generation tools can be utilized to realize the domain dissection in an efficient way. Therewith, geometries such as the hopper of Figure 1 that are non-convex in shape and have holes can be generated. Within a particle cluster, the size distribution of the particles may differ in order to obtain a high geometrical closeness to the desired geometry and to represent the special features of the geometry. To establish the adhesion between the particles within the cluster, the particles are coupled via their faces in a penalty approach. Compressive or tensile stress is applied to the bonded faces and results in nodal forces. Those forces aim to keep the bonded faces coplanar and aim to prevent node-pair disparities at any time. Exceeding the compressive or tensile strength of the material triggers a decomposition of the bond and initiates fracture [7].

The tetrahedral particles are considered as elastically deformable whereas the deformations and resulting stresses in each particle are described similar to a linear described tetrahedral finite element. DEM neighborhood search includes two steps. First step is a rough but fast neighborhood search based on oriented bounding boxes. The following second step, the numerically expensive contact evaluation, has to be capable to handle non-spherical non-convex and deformable particles. Therefore, contact algorithms used by the family of spherical particles or the family of particles defined by inequalities cannot be used. The algorithm to detect contact between two tetrahedral particles defined as in this contribution is based on an optimization problem. The objective function describes the closest distance of a point to all faces of the two tetrahedra. The contact point results from a minimum of the objective function for the domain. Therewith, the contact algorithm is capable to find the contact point independent of the contact type such as vertex-vertex, vertex-edge, vertex-face, edge-edge, edge-face, or face-face.

This approach is used for the simulation shown in Figure 1. Clusters of deformable tetrahedra are used to represent cubes that flow through a hopper. Due to the nearly symmetrical initial setup the cubes restrain each other from moving through the hole, before they finally move through the hole one by one.

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Contact models for discrete element simulation of the initial powder compaction in a hot pressing process

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Summary: This paper presents discrete element studies of a powder compaction process using two different contact models. Performance of the elastic Hertz–Mindlin–Deresiewicz model has been compared with that of the plastic Storåkers model. The results of laboratory tests of the die compaction of the NiAl powder have been used to validate numerical results.

Introduction

The discrete element method (DEM) has been successfully employed to modelling powder metallurgy processes involving sintering, cf. [1–3]. The sintering in a hot pressing process is performed under uniaxial pressure applied before heating the powder in a die. The present paper is focused on modelling the initial compaction in this process.

Contact models

A number of contact models for powder compaction have been developed within the framework of the discrete element method [4]. A model assuming rigid plastic behaviour according to the Hollomon stress-strain curve has been developed by Storåkers et al. [5, 6]. The hot pressing is performed with a relatively low pressure (up to 50 MPa) therefore we have checked if the elastic model is suitable to represent properly densification mechanisms at these conditions. Compression of two equal particles with radii $R = 10\ \mu\text{m}$ (Figure 1(a)) typical for the real powder particles has been analysed. Figure 1(b) shows comparison of contact force curves as functions of particle indentation for the elastic Hertz model and the plastic Storåkers model with different values of the hardening exponent m.

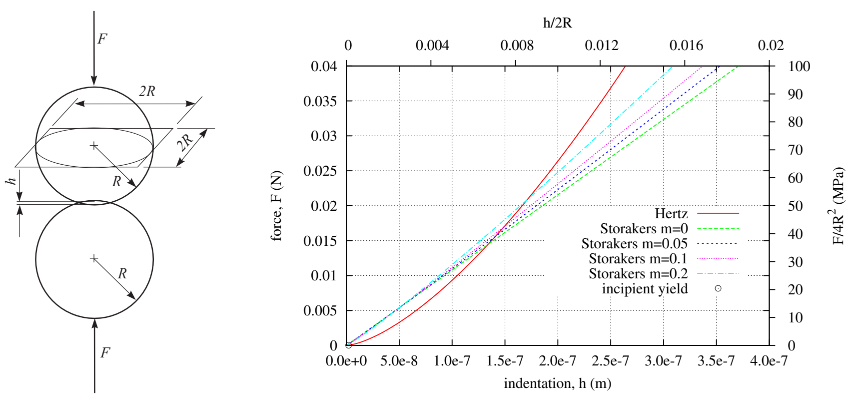
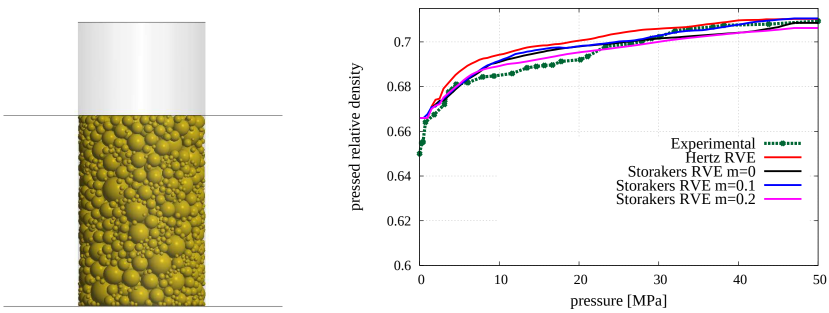


Figure 1: Compression of two equal particles: (a) problem definition, (b) contact force vs. indentation curves.

Simulation of powder compaction in a cylindrical die

The die compaction process of the intermetallic NiAl powder has been investigated experimentally and studied numerically. Simulations have been performed keeping the real size and size distribution of the powder particles and using a reduced cylindrical specimen and shown in Figure 2(a). The curves showing the relative density as functions of the applied pressure are given in Figure 2(b). The numerical results have been obtained for the Hertz and Storåkers models with zero friction conditions. The numerical results are compared with the experimental data showing quite a good agreement. Differences between the numerical results obtained with different models are relatively small. This is because the differences between the force–indentation curves predicted by the compared models in the range of contact pressure in our studies are relatively small, cf. Figure 1(b).



(a) (b)

Figure 2: Simulation of powder compaction in a cylindrical die: (a) DEM model, (b) relative density as a function of pressure – comparison of the numerical results for zero friction with the experimental data.

Acknowledgement

The results presented in this paper have been obtained within the project funded by the Polish National Science Centre awarded by decision numbers DEC-2013/11/B/ST8/03287.

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PERMON's scalable quadratic programming solvers for contact problems based on TFETI

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Summary: PERMON toolbox makes use of theoretical results in discretization techniques, quadratic programming algorithms, and domain decomposition methods. It is built on top of the PETSc framework for numerical computations. The inequality constrained quadratic programming problems can be solved by our PermonQP package in combination with PermonFLLOP employing FETI methods.

Introduction

We shall present our new software called PERMON (Parallel, Efficient, Robust, Modular, Object-oriented, Numerical) toolbox [3]. It aims at massively parallel solution of problems of linear elasticity, contact problems with friction, elasto-plasticity, shape optimization and others. It features special algorithms for quadratic programming with generalized constraints (convex separable constraints such as elliptical ones arising from Coulomb friction), and domain decomposition methods (DDM) which allow efficient and robust utilization of parallel computers up to tens of thousands processor cores and billions of unknowns. By these means we are able to shorten the solution time or even to solve problems whose huge dimension make them unsolvable on conventional personal computers.

PERMON is based on PETSc and combines aforementioned DDM and QP algorithms. The core solver layer consists of the PermonQP package for QP and its PermonFLLOP extension for FETI. The main idea of PermonQP is separation of QP problems, their transformations and solvers. A QP transformation derives a new QP from the given QP, thus, sort of doubly linked list is generated where every node is a QP.

PermonFLLOP implements a Total-FETI (TFETI) variant of the Finite Element Tearing and Interconnecting (FETI) DDM. FETI was originally introduced by Farhat and Roux in early 90's. It belongs to non-overlapping methods, it combines iterative and direct sparse linear solvers, and it allows highly accurate computations scaling up to tens of thousands of processors. The TFETI variant was proposed by Dostal et al. [1]. It uses Lagrange multipliers to enforce Dirichlet boundary conditions, which leads to simpler, yet more efficient implementation.

Resulting QP problems are solved by solvers within PermonQP: MPRGP (Modified Proportioning with Reduced Gradient Projections) and SMALBE (SemiMonotonic Augmented Lagrangian algorithm for Bound and Equality constraints) algorithms, proposed by Dostal [2]. The algorithms have the rate of convergence in terms of bounds on the spectrum of the Hessian matrices. In combination with TFETI, these algorithms were proven to enjoy both numerical and parallel scalability for solution of the contact problems (frictionless, Tresca friction, transient) of elasticity.

Numerical experiments demonstrating the performance including the highlights with model and engineering problems will be presented at the conference.

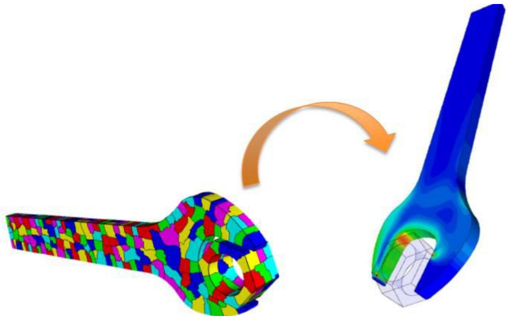

Figure 1: Engineering contact problem – its domain decomposition and solution.

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A new procedure for the solution of quasistatic frictional contact problems by the symmetric Galerkin boundary element method and quadratic programming

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Summary: A procedure for the numerical solution of two-dimensional quasistatic frictional contact problems between linear elastic and visco-elastic solids is proposed, implemented and tested. Linearly responding normal compliance and Coulomb friction are assumed in the present contact model. The numerical solution is based on the evaluation of the evolution of energies during the loading process. It includes a semi-implicit time discretisation and a spatial discretisation by the symmetric Galerkin boundary element method (SGBEM), leading to a minimization problem with a non-smooth cost functional in each time step. A transformation of the contact quantities provides a quadratic-programming structure of this minimization problem. Eventually a very efficient and accurate numerical procedure for such contact problems is implemented in a computational code. Some interesting frictional contact problems with non-proportional loading and possible solution jumps are solved and discussed.

Numerical procedure

The procedure for the numerical solution of two-dimensional frictional contact problems which was recently developed, implemented and tested in [3, 5] is briefly described in the present work. First a classical formulation of the evolution boundary-value contact problems is introduced. Then, this formulation is rewritten in the form of a weak formulation by using a convex stored energy functional and a non-smooth convex potential of dissipative forces.

The numerical procedure adopted here considers time and spatial discretisation separately. The time-stepping scheme is based on a semi-implicit recursive scheme in time.

In many contact problems the nonlinearities are restricted to the boundaries, e.g., when the solids in contact are linear elastic or visco-elastic and homogeneous, which allows a straightforward application of the Boundary Element Method (BEM), see [1] for some classical references. The present work is limited to such cases, where the BEM can be an efficient and very accurate alternative to the Finite Element Method (FEM). The consideration of linear visco-elastic solids is based on the approach developed in [2].

The spatial discretisation is carried out here by the symmetric Galerkin boundary element method (SGBEM) similarly as in [4]. An advantage of the SGBEM in comparison with the classical collocational BEM is that it leads to symmetric linear systems and is more amenable for energetic formulations.

Finally, a transformation of the non-smooth incremental problems to get the quadratic programming (QP) structure is applied. The conjugate gradient based algorithms with bound constraints are used to solve quadratic programming problems.

Numerical stability (in the sense of a-priori estimates uniform with respect to the time and space discretisation) and convergence towards solutions of the original continuous problem can be rigorously shown, cf. [5, 6].

Examples

The numerical solutions of the frictional contact problems presented illustrate the capability of the implemented computational code to effectively deal with non-proportional loading and possible solution jumps. An example of a solved non-trivial contact problem is shown in Fig. 1.

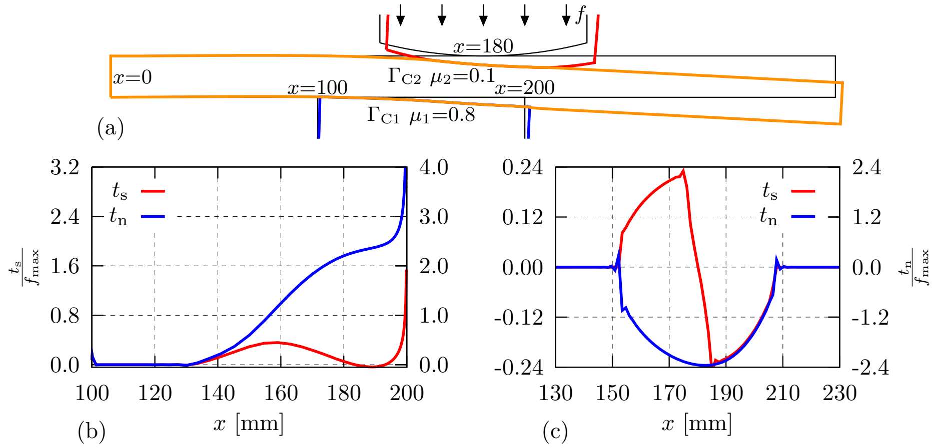

Figure 1: A three-body contact problem loaded by incrementally increased f up to $f_{\max}$=100MPa. Results for $f_{\max}$: (a) deformed and undeformed configurations; normal t_{n} and tangential t_{s} contact stresses along (b) Γ_{C1} and (c) Γ_{C2}.

Acknowledgements: VEGA 1/0078/16 (Slovakia), MAT2012-37387 (Spain)

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A variational analysis of a dynamic contact problem with Coulomb friction

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Summary: This work is concerned with the study of a dynamic viscoelastic problem with general relaxed unilateral contact conditions and Coulomb friction. This problem constitutes a unified approach to study various interface models and its analysis is based on approximation results for an evolution implicit variational inequality.

Introduction

The aim of this work is to generalize some recent results presented in [1, 2] for the dynamic contact between two viscoelastic bodies. These results extend the method used in [3] to solve an elastostatic contact problem. This approach enables to study some complex interface models, describing the coupling between relaxed unilateral contact conditions, adhesion and Coulomb friction. Based on the variational formulation given in [4], quasistatic elastic problems with unilateral contact conditions and Coulomb friction law were analyzed in [5, 6] and an adhesion model was investigated in [7, 8]. Dynamic viscoelastic contact problems with nonlocal friction laws were solved in [9–11]. Using the Clarke subdifferential, various contact problems can be analyzed by using the theory of hemivariational inequalities, see [12–14] and references therein, while the method presented here is more direct and well adapted to the treatment of the Coulomb friction law.

Classical and variational formulations

We consider two viscoelastic bodies, characterized by a Kelvin-Voigt constitutive law, which occupy the reference domains Ω^α of $\mathbb{R}^d$, $d = 2$ or 3, with Lipschitz boundaries $\Gamma^\alpha = \partial\Omega^\alpha$, $\alpha = 1, 2$. Let Γ^α_U, Γ^α_F and Γ^α_C be three disjoint parts of Γ^α such that $\Gamma^\alpha = \overline{\Gamma}^\alpha_U \cup \overline{\Gamma}^\alpha_F \cup \overline{\Gamma}^\alpha_C$, $\alpha = 1, 2$. Let $\boldsymbol{\sigma}^\alpha$ and $\boldsymbol{\varepsilon}(\boldsymbol{u}^\alpha)$ be the stress tensor and the infinitesimal strain tensor corresponding to the displacement vector field $\boldsymbol{u}^\alpha$ in Ω^α, $\alpha = 1, 2$. Suppose that the solids can be in contact between the potential contact surfaces Γ^1_C and Γ^2_C which are parametrized by two C^1 functions defined on a subset Ξ of $\mathbb{R}^{d-1}$. Consider the following dynamic viscoelastic contact problem with Coulomb friction.

Problem $\boldsymbol{P}$: Find the displacement vector $\boldsymbol{u} = (\boldsymbol{u}^1, \boldsymbol{u}^2)$ such that $\boldsymbol{u}(0) = \boldsymbol{u}_0$, $\dot{\boldsymbol{u}}(0) = \boldsymbol{u}_1$ and, for all $t \in (0, T)$,

$$\ddot{\boldsymbol{u}}^\alpha - \operatorname{div} \boldsymbol{\sigma}^\alpha(\boldsymbol{u}^\alpha, \dot{\boldsymbol{u}}^\alpha) = \boldsymbol{f}^\alpha_1, \quad \boldsymbol{\sigma}^\alpha(\boldsymbol{u}^\alpha, \dot{\boldsymbol{u}}^\alpha) = \boldsymbol{\mathcal{A}}^\alpha \boldsymbol{\varepsilon}(\boldsymbol{u}^\alpha) + \boldsymbol{\mathcal{B}}^\alpha \boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}^\alpha) \ \text{ in } \ \Omega^\alpha, \quad \alpha = 1, 2,$$

$$\boldsymbol{u}^\alpha = \boldsymbol{0} \ \text{ on } \ \Gamma^\alpha_U, \ \boldsymbol{\sigma}^\alpha \boldsymbol{n}^\alpha = \boldsymbol{f}^\alpha_2 \ \text{ on } \ \Gamma^\alpha_F, \ \alpha = 1, 2, \quad \boldsymbol{\sigma}^1 \boldsymbol{n}^1 + \boldsymbol{\sigma}^2 \boldsymbol{n}^2 = \boldsymbol{0} \ \text{ in } \ \Xi,$$

$$\sigma^1_N \in \Pi(u_N, \dot{u}_N) \ \text{ in } \ \Xi,$$

$$|\boldsymbol{\sigma}^1_T| \le \mu(\dot{\boldsymbol{u}}_T)\, |\sigma^1_N| \ \text{ and } \ \dot{\boldsymbol{u}}_T \ne \boldsymbol{0} \Rightarrow \boldsymbol{\sigma}^1_T = -\mu(\dot{\boldsymbol{u}}_T) |\sigma^1_N| \frac{\dot{\boldsymbol{u}}_T}{|\dot{\boldsymbol{u}}_T|} \ \text{ in } \ \Xi,$$

where u_N, $\boldsymbol{u}_T$, σ^1_N, $\boldsymbol{\sigma}^1_T$ denote the normal and tangential components of the relative displacement field and of the stress vector $\boldsymbol{\sigma}^1 \boldsymbol{n}^1$, respectively, Π is a given multivalued mapping that describes the contact condition depending on the gap and on the normal velocity and $\mu = \mu(\dot{\boldsymbol{u}}_T)$ is the slip dependent coefficient of friction. Various contact conditions can be obtained as particular cases of this general formulation. The variational formulation is a two-field problem that consists in an evolution implicit variational inequality having as unknowns the displacement field and a Lagrange multiplier corresponding to σ_N^1. We prove the existence of variational solutions and some estimates established for these solutions enable to investigate the approximation of the corresponding problem with Signorini's conditions and Coulomb friction law.

Acknowledgements

The partial support of this work by "Laboratoire Européen Associé CNRS Franco-Roumain de Mathématiques et Modélisation" is gratefully acknowledged.

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History-dependent Variational-Hemivariational Inequalities with Applications to Dynamic Contact Problem in Viscoelasticity

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Summary: We study an initial boundary value problem which describes a model of a viscoelastic body in contact with a foundation. The contact process is assumed to be dynamic and the friction is described by subdifferential boundary conditions. Both the constitutive law and the contact condition involve memory operators.

Contact model

The physical setting of the contact problem is as follows. A deformable viscoelastic body occupies a set $\Omega \subset \mathbb{R}^d$, $d = 2, 3$. The volume forces and surface tractions depend on time and act on the body. We consider the dynamic process on the finite time interval $[0, T]$. The boundary $\Gamma = \partial\Omega$ is supposed to be Lipschitz continuous and to be composed of three parts Γ_D, Γ_N and Γ_C which are mutually disjoint, and the measure of Γ_D is positive. The unit outward normal vector on Γ is denoted by $\boldsymbol{\nu}$. We suppose that the body is clamped on part Γ_D, volume forces of density $\boldsymbol{f}_0$ act in Ω, and surface tractions of density $\boldsymbol{f}_N$ are applied on Γ_N. The body may come in contact with an obstacle over the potential contact surface Γ_C. We put $Q = \Omega \times (0, T)$, $\Sigma_D = \Gamma_D \times (0, T)$, $\Sigma_N = \Gamma_N \times (0, T)$ and $\Sigma_C = \Gamma_C \times (0, T)$. We denote by $\mathbb{S}^d$ the space of $d \times d$ symmetric matrices, and we do not indicate explicitly the dependence of functions on the spatial variable $\boldsymbol{x} \in \Omega$.

We denote by $\boldsymbol{u} \colon Q \to \mathbb{R}^d$ the displacement vector, by $\boldsymbol{\sigma} \colon Q \to \mathbb{S}^d$ the stress tensor and by $\boldsymbol{\varepsilon}(\boldsymbol{u}) = (\varepsilon_{ij}(\boldsymbol{u}))$ the linearized (small) strain tensor, where $i, j = 1, \ldots, d$. Recall that the components of the linearized strain tensor are given by $\boldsymbol{\varepsilon}(\boldsymbol{u}) = 1/2(u_{i,j} + u_{j,i})$, where $u_{i,j} = \partial u_i / \partial x_j$. By u_ν, σ_ν and $\boldsymbol{u}_\tau$, $\boldsymbol{\sigma}_\tau$ we denote the normal and tangential components $\boldsymbol{u}$ and $\boldsymbol{\sigma}$, respectively. The classical formulation of the problem reads as follows.

Problem $\mathcal{P}$. *Find a displacement field $\boldsymbol{u} \colon Q \to \mathbb{R}^d$ and a stress field $\boldsymbol{\sigma} \colon Q \to \mathbb{S}^d$ such that*

$$\boldsymbol{u}''(t) - \operatorname{Div} \boldsymbol{\sigma}(t) = \boldsymbol{f}_0(t) \qquad \text{in} \quad Q$$

$$\boldsymbol{\sigma}(t) = \mathcal{A}(t, \boldsymbol{\varepsilon}(\boldsymbol{u}'(t))) + \mathcal{B}(t, \boldsymbol{\varepsilon}(\boldsymbol{u}(t))) + \int_0^t \mathcal{K}(t-s, \boldsymbol{\varepsilon}(\boldsymbol{u}'(s)))\, ds \qquad \text{in} \quad Q,$$

$$\boldsymbol{u}(t) = \boldsymbol{0} \qquad \text{on} \quad \Sigma_D$$

$$\boldsymbol{\sigma}(t)\boldsymbol{\nu} = \boldsymbol{f}_N(t) \qquad \text{on} \quad \Sigma_N$$

$$-\sigma_\nu(t) \in \partial j_\nu(t, u'_\nu(t)) \qquad \text{on} \quad \Sigma_C$$

$$-\boldsymbol{\sigma}_\tau(t) \in h(u_\nu(t))\, \partial\psi(\boldsymbol{u}'_\tau(t)) \qquad \text{on} \quad \Sigma_C$$

$$\boldsymbol{u}(0) = \boldsymbol{u}_0, \quad \boldsymbol{u}'(0) = \boldsymbol{v}_0 \qquad \text{in} \quad \Omega.$$

We note that in the viscoelastic constitutive law with long memory, $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{K}$ are nonlinear time dependent viscosity, elasticity and relaxation operators, respectively. The multivalued conditions on Σ_C represent the contact and friction conditions in which j_ν, h and ψ are given functions. The function j_ν is locally Lipschitz in the second variable and ∂j_ν denotes its Clarke subdifferential while the function ψ is convex in the second variable and $\partial\psi$ stands for its convex subdifferential. The examples of contact condition cover various variants of the nonmonotone normal damped response condition and are presented in [1, 2]. The friction condition incorporates several conditions met in the literature, the simplest example leads to the Coulomb law of dry friction. Finally, $\boldsymbol{u}_0$ and $\boldsymbol{v}_0$ denote the initial displacement and the initial velocity, respectively.

Main result

We introduce the spaces $V = \{\, \boldsymbol{v} \in H^1(\Omega;\mathbb{R}^d) \mid \boldsymbol{v} = 0 \ \text{ on } \Gamma_D \,\}$ and $\mathcal{H} = L^2(\Omega;\mathbb{S}^d)$. The variational formulation of Problem $\mathcal{P}$ reads as follows.

Problem $\mathcal{P}_V$. *Find a displacement field $\boldsymbol{u}\colon Q \to \mathbb{R}^d$ and a stress field $\boldsymbol{\sigma}\colon Q \to \mathbb{S}^d$ such that*

$$\boldsymbol{\sigma}(t) = \mathcal{A}(t, \boldsymbol{\varepsilon}(\boldsymbol{u}'(t))) + \mathcal{B}(t, \boldsymbol{\varepsilon}(\boldsymbol{u}(t))) + \int_0^t \mathcal{K}(t-s, \boldsymbol{\varepsilon}(\boldsymbol{u}'(s)))\, ds \ \text{ a.e. } \ t \in (0,T),$$

$$\begin{aligned} &\langle \boldsymbol{u}''(t), \boldsymbol{v} - \boldsymbol{u}'(t)\rangle_{V^*\times V} + (\boldsymbol{\sigma}(t), \boldsymbol{\varepsilon}(\boldsymbol{v}) - \boldsymbol{\varepsilon}(\boldsymbol{u}'(t)))_{\mathcal{H}} + \\ &\qquad + \int_{\Gamma_C} j_\nu^0(t, u_\nu'(t); v_\nu - u_\nu'(t))\, d\Gamma + \int_{\Gamma_C} h(u_\nu(t))(\psi(\boldsymbol{v}_\tau) - \psi(\boldsymbol{u}_\tau'(t)))\, d\Gamma \geq \\ &\qquad \geq \langle \boldsymbol{f}(t), \boldsymbol{v}\rangle_{V^*\times V} \ \text{ for all } \ \boldsymbol{v} \in V, \ \text{a.e. } t \in (0,T), \end{aligned}$$

$$\boldsymbol{u}(0) = \boldsymbol{u}_0, \qquad \boldsymbol{u}'(0) = \boldsymbol{v}_0,$$

where j_ν^0 denotes the generalized directional derivative of a locally Lipschitz function in the sense of Clarke. Such kind of problems leads to a new and nonstandard mathematical model which consists of variational-hemivariational inequality of first order with history-dependent operators. Our main result in the study of Problem $\mathcal{P}_V$ provides hypotheses on the data under which the contact problem has a weak solution. Moreover, under a smallness condition involving the constants appearing in the problem, we prove that the problem has a unique local in time weak solution. Our main results are obtained by combining a fixed point argument and a recent result for evolution subdifferential inclusion provided in [2, 3].

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Existence and uniqueness results for dynamic thermo-elsato-plastic contact problems

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Summary: We are interested in a motion of a one-dimensional visco-elasto-plastic body in contact with an elasto-plastic obstacle and undergoing thermal expansion. The existence result for this thermodynamically consistent problem follows from some a priori estimates established for the space discretized problem while the uniqueness result comes from the Lipschitz continuity of the nonlinearities.

Introduction

The aim of this work consists to give some new mathematical results on existence and uniqueness for thermo-elasto-plastic dynamical contact problems. The situations involving contact abound in industry, especially in engines or transmissions. For this reason a considerable engineering and mathematical literature deals with dynamic and quasi–static frictional contact problems. This work is a continuation of [4] where contact problems are reformulated as partial differential equations with hysteresis operators in the bulk and on the boundary, a one dimensional thermomechanically model taking into account the exchange between different types of energy in an oscillating visco-elasto-plastic body in contact with an elasto–plastic obstacle is proposed and analyzed.

We consider an elasto-plastic bar of length L vibrating longitudinally; on the one end, the bar is free to move as long as it does not hit a material obstacle, while on the other end, a force is applied. We begin by rewritten the problem in accordance with the formalism of hysteresis operators as solution operators of the underlying variational inequalities. The displacement $u(x,t)$ at time t of the material point of spatial coordinate $x \in (0,L)$ is one of the unknowns of the problem and σ is the σ_{11} component of the stress tensor. Therefore the motion is governed by the following differential equation:

$$\rho \frac{\partial^2 u}{\partial t^2}(x,t) - \frac{\partial \sigma}{\partial x}(x,t) = 0, \tag{1}$$

where $\rho > 0$ is the mass density. Furthermore, the stress σ satisfies the following constitutive equation:

$$\sigma(x,t) \stackrel{\text{def}}{=} \mathcal{P}[\mathrm{e}](x,t) + \nu \frac{\partial \mathrm{e}}{\partial t}(x,t) - \beta(\theta(x,t) - \theta^{\text{ref}}) \quad \text{and} \quad \mathrm{e}(x,t) \stackrel{\text{def}}{=} \frac{\partial u}{\partial x}(x,t), \tag{2}$$

where e is the e_{11} component of the strain tensor, $\theta(x,t) > 0$ is the absolute temperature which is one of the unknowns of the problem, $\nu > 0$ is the viscosity modulus, $\beta \in \mathbb{R}$ is the thermal expansion coefficient, and $\theta^{\text{ref}} > 0$ is a given referential temperature. The symbol $\mathcal{P}$ denotes a constitutive operator of elasto-plasticity satisfying the following identity:

$$\mathcal{P}[\mathrm{e}](x,t) = \lambda \mathrm{e}(x,t) + \mathcal{P}_0[\mathrm{e}](x,t), \tag{3}$$

where $\mathcal{P}_0[\mathrm{e}]$ corresponds to the plastic stress component with yield point $r > 0$ and elasticity domain $K \stackrel{\text{def}}{=} [-r, r]$ and satisfies

$$\begin{cases} \mathcal{P}_0[\mathrm{e}](\cdot,t) \in K & \text{for all } t \in [0,T], \\ \mathcal{P}_0[\mathrm{e}](\cdot,0) = \mathrm{Proj}_K(\mathbb{E}\mathrm{e}(\cdot,0)), & \\ (\mathbb{E}\mathrm{e}_t(\cdot,t) - \mathcal{P}_{0,t}[\mathrm{e}](\cdot,t))(\mathcal{P}_0[\mathrm{e}](\cdot,t) - y) \geq 0 & \text{a.e. for all } y \in K. \end{cases} \tag{4}$$

Here Proj_K is the projection onto K, the constant $\mathbb{E} > 0$ is the elasticity modulus, and $\lambda > 0$ is the kinematic hardening modulus.

The sovability for this thermodynamically consistent problem is obtained under some appropriate regularity on the data. More precisely, a space discretization is introduced and some a priori estimates are obtained by using both the classical energy estimate and more specific techniques like the Dafermos estimate [2] and some Sobolev interpolation inequalities [1, 3] leading to the existence result. The uniqueness result follows from the Lipschitz continuity of the nonlinearities.

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Models of Elastic Shells in Contact with a Rigid Foundation

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Summary: By using asymptotic analysis arguments, we obtain different models for elastic shells in contact with a rigid foundation. We also advance some convergence results which justify the procedure and assumptions made.

Introduction

In solid mechanics, the obtention of models for rods, beams, plates and shells is based on *a priori* hypotheses on the displacement and/or stress fields which, upon substitution in the three-dimensional equilibrium and constitutive equations, lead to useful simplifications. Nevertheless, from both constitutive and geometrical point of views, there is a need to justify the validity of most of the models obtained in this way.

For this reason a considerable effort has been made in the past decades by many authors in order to derive new models and justify the existing ones by using the asymptotic expansion method, whose foundations can be found in [1]. For example, a complete theory regarding elastic shells can be found in [2], where models are presented for elliptic membranes, generalized membranes and flexural shells.

Objectives and Methodology

In this work, we obtain and justify models for elastic shells in contact with a rigid foundation. The contact is modelled by using the classical unilateral Signorini condition. The procedure which we follow consists in the following. First we formulate a variational formulation of the mechanical problem in curvilinear coordinates, defined in the set $\bar{\Omega}^\varepsilon = \bar{\omega} \times [-\varepsilon, \varepsilon]$, where ω is a two-dimensional domain (the middle section) and ε is an *small* parameter, the thickness of the shell. We identify the shell with the set $\Theta(\bar{\Omega}^\varepsilon)$, where $\Theta : \bar{\Omega}^\varepsilon \subset \mathbb{R}^3 \to \mathbb{R}^3$ is the so-called canonical mapping (a sufficiently smooth and injective mapping). Thus, we obtain a variational inequality whose solution (the displacements field) is in a convex subset of a suitable Hilbert space. Then, we perform a change of variable in order to obtain an equivalent problem posed on a domain independent of ε, obtaining the so-called scaled variational problem. Then, we assume an asymptotic expansion of the scaled displacements and proceed to analyse the resulting system of variational inequalities after cancelling terms of the different orders. We obtain characterizations of the first order terms as the unique solution of two-dimensional variational inequalities. Depending on such aspects as the geometry of the middle surface, S, of the shell, the part of the lateral boundary where the conditions of place are imposed or the order of the applied forces, we obtain different models, namely *membranes* and *flexural shells*. We then re-scale the two-dimensional variational inequalities in order to obtain models with a physical meaning. We finally comment on the required results of convergence which give a rigourous justification of these models.

The models

For the sake of definiteness, we provide here one of the models that we obtain. Assume that the middle surface of the shell, S, is elliptic and the entire lateral boundary is fixed. Assume that the vector field $\boldsymbol{u}^\varepsilon = (u_i^\varepsilon)$ (associated to a displacement field $\tilde{\boldsymbol{u}}^\varepsilon = u_i^\varepsilon \boldsymbol{g}^{i,\varepsilon}$, where $\{\boldsymbol{g}^{i,\varepsilon}\}$ is the contravariant basis of the tangent space to $\Theta(\bar{\Omega}^\varepsilon)$) has an asymptotic expansion of the form $\boldsymbol{u}^\varepsilon = \boldsymbol{u}^0 + \varepsilon \boldsymbol{u}^1 + \varepsilon^2 \boldsymbol{u}^2 + \ldots$. Then, we can show that the zero-th order term $\boldsymbol{u}^0$ can be identified with the solution of the following two-dimensional variational inequality:

$$\boldsymbol{\xi}^\varepsilon = (\xi_i^\varepsilon) \in K_M(\omega) = \{\boldsymbol{\eta}^\varepsilon = (\eta_i^\varepsilon) \in H_0^1(\omega) \times H_0^1(\omega) \times L^2(\omega),\ \eta_3^\varepsilon \geq 0\},$$
$$\varepsilon \int_\omega a^{\alpha\beta\sigma\tau,\varepsilon} \gamma_{\sigma\tau}(\boldsymbol{\xi}^\varepsilon) \gamma_{\alpha\beta}(\boldsymbol{\eta} - \boldsymbol{\xi}^\varepsilon) \sqrt{a} dy \geq \int_\omega p^{i,\varepsilon}(\eta_i - \xi_i^\varepsilon) \sqrt{a} dy \ \forall \boldsymbol{\eta} = (\eta_i) \in K_M(\omega),$$

where $\gamma_{\alpha\beta}(\boldsymbol{\eta})$ are the covariant components of the linearized change of metric tensor associated with a displacement $\tilde{\boldsymbol{\eta}} = \eta_i \boldsymbol{a}^i$ of the middle surface S (here $\{\boldsymbol{a}^i\}$ represents the contravariant basis of the tangent plane to S), and

$$p^{i,\varepsilon} = \int_{-\varepsilon}^{\varepsilon} f^{i,\varepsilon} dx_3^\varepsilon + h_+^{i,\varepsilon} + h_-^{i,\varepsilon}, \quad \text{with } h_\pm^{i,\varepsilon} = h^{i,\varepsilon}(\cdot, \pm\varepsilon),$$

is the term representing the action of the external forces $\boldsymbol{f}^\varepsilon = (f^{i,\varepsilon})$ and tractions $\boldsymbol{h}^\varepsilon = (h^{i,\varepsilon})$. Also, the contravariant components of the fourth-order two-dimensional tensor $(a^{\alpha\beta\sigma\tau,\varepsilon})$ are defined as follows:

$$a^{\alpha\beta\sigma\tau,\varepsilon} = \frac{4\lambda^\varepsilon \mu^\varepsilon}{\lambda^\varepsilon + 2\mu^\varepsilon} a^{\alpha\beta} a^{\sigma\tau} + 2\mu^\varepsilon (a^{\alpha\sigma} a^{\beta\tau} + a^{\alpha\tau} a^{\beta\sigma}),$$

where $a^{\alpha\beta} = \boldsymbol{a}^\alpha \cdot \boldsymbol{a}^\beta$ are the contravariant components of the metric tensor of S and $\lambda^\varepsilon, \mu^\varepsilon$ are the Lamé coefficients of the material.

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Dynamic frictional contact of a thermoviscoelastic body with an obstacle

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Summary: We study the dynamic behavior of a thermoviscoelastic body that may come into frictional contact with an obstacle. We employ the Kelvin-Voigt constitutive law and the subdifferential boundary conditions, which include the thermal effects. The existence of weak solutions is proved by using a surjectivity result for operators of pseudomonotone type.

Physical setting

A deformable thermoviscoelastic body occupies a bounded domain $\Omega \subset \mathbb{R}^d$. The boundary $\Gamma = \partial\Omega$ is Lipschitz continuous and composed of three mutually disjoint parts Γ_D, Γ_N and Γ_C such that meas$(\Gamma_D) > 0$ with $\boldsymbol{\nu}$ the unit outward normal vector on Γ_D. We are interested in a mathematical model that describes the evolution of the mechanical state of the body and its temperature during the time interval $[0, T]$. We suppose that the body is clamped on Γ_D, the volume forces of density $\boldsymbol{f_0}$ act in Ω and the surface tractions of density $\boldsymbol{f_N}$ are applied on Γ_N. Moreover, the body is subjected to a heat source term per unit volume g and it comes in contact with an obstacle over the contact surface Γ_C. By u'_ν, σ_ν and $\boldsymbol{u}'_\tau$, $\boldsymbol{\sigma}_\tau$ we denote the normal and tangential components of $\boldsymbol{u}'$ and $\boldsymbol{\sigma}$, respectively. We suppress the explicit dependence of functions on the spatial variable $\boldsymbol{x}$, or both $\boldsymbol{x}$ and t. We assume that the temperature changes accompanying the deformations are small and they do not produce any changes in the material parameters which are regarded temperature independent. Assuming small displacements, the classical formulation of the problem can be stated as follows.

Problem $\mathcal{P}$:
Find a displacement field $u: \Omega \times (0, T) \to \mathbb{R}^d$ and a temperature $\theta: \Omega \times (0, T) \to \mathbb{R}$ such that

$$\begin{aligned}
u''_i - \partial_j \sigma_{ij} = f_{0i} \quad & \text{in} \quad \Omega \times (0,T) \\
\sigma_{ij} = a_{ijkl}\partial_l u'_k + b_{ijkl}\partial_l u_k - c_{ij}\theta \quad & \text{in} \quad \Omega \times (0,T) \\
\theta' + \partial_i q_i + c_{ij}\partial_j u'_i = g \quad & \text{in} \quad \Omega \times (0,T) \\
q_i = -k_{ij}\partial_j \theta \quad & \text{in} \quad \Omega \times (0,T) \\
\boldsymbol{u} = \boldsymbol{0} \quad & \text{on} \quad \Gamma_D \times (0,T) \\
\boldsymbol{\sigma\nu} = \boldsymbol{f}_N \quad & \text{on} \quad \Gamma_N \times (0,T) \\
\theta = 0 \quad & \text{on} \quad (\Gamma_D \cup \Gamma_N) \times (0,T) \\
-\sigma_\nu \in p(\theta)\, \partial j_\nu(u'_\nu) \quad & \text{on} \quad \Gamma_C \times (0,T) \\
-\boldsymbol{\sigma}_\tau \in q(\theta)\, \partial j_\tau(\boldsymbol{u}'_\tau) \quad & \text{on} \quad \Gamma_C \times (0,T) \\
-\frac{\partial\theta}{\partial\nu_K} \in r(\boldsymbol{u}'_\tau)\, \partial j(\theta) \quad & \text{on} \quad \Gamma_C \times (0,T) \\
\boldsymbol{u}(0) = \boldsymbol{u}_0, \quad \boldsymbol{u}'(0) = \boldsymbol{v}_0, \quad \theta(0) = \theta_0 \quad & \text{in} \quad \Omega
\end{aligned}$$

where $i, j, k, l = 1, \ldots, d$, the repeated index convention is used and $\partial_j = \partial/\partial x_j$. Moreover, $\{a_{ijkl}\}$, $\{b_{ijkl}\}$ are the viscosity and elasticity tensors, respectively, $\{c_{ij}\}$ is the coefficients of thermal expansion, and p, q and r are prescribed functions.

Our main interest lies in the multivalued conditions on $\Gamma_C \times [0, T]$. The relations between the normal stress σ_ν and the normal velocity u'_ν, the tangential force $\boldsymbol{\sigma}_\tau$ and the tangential velocity $\boldsymbol{u}'_\tau$, and the boundary temperature θ and the heat flux vector $q_i \nu_i = -\frac{\partial \theta}{\partial \nu_K} := -k_{ij} \partial_j \theta \nu_i$ are described by the Clarke subdifferential of locally Lipshitz functions j_ν, j_τ and j, respectively. The examples which illustrate these boundary conditions are presented in [1].

Main result

The variational formulation of Problem $\mathcal{P}$ leads to a system of coupled hemivariational inequalities: the hemivariational inequality of the second order for a displacement and the hemivariational inequality of the first order for a temperature. Our main result in the study of variational problem provides hypotheses on the data under which the contact problem has a weak solution. We formulate a system of coupled evolution inclusions related to the system of hemivariational inequalities and apply the surjectivity result for the sum of a linear, densely defined and maximal monotone operator L and the bounded, coercive and L-pseudomonotone operator.

We stress that our problem extends the contact model considered in [2]. The novelty of current model consists of the subdifferential boundary conditions which include the thermal effects in contact and friction conditions as well as the frictional heating effects in the boundary condition for the temperature on Γ_C. Taking these effects into account is essential in many applications.

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Interfacial stiffnesses of rough surfaces in no-sliding contact via an imperfect interface approach

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Summary: A spring-like micromechanical contact model is proposed, aiming to describe the mechanical behavior of two rough surfaces in no-sliding contact under a closure pressure. Tangential and normal equivalent contact stiffnesses are consistently derived via the Imperfect Interface Approach (IIA).

Abstract

Analytical and numerical modeling of contact problems related to rough surfaces can be surely considered as an open and challenging research topic, strictly associated to many applications in different engineering fields. From a computational point of view, it is possible to identify a class of modeling problems in which it is neither possible nor convenient to account for a fine and detailed description of the contact regions, although local contact features may strongly affect the overall mechanical response for the problem under investigation. In these cases, a possible strategy is based on modeling contact scenarios by introducing suitable stiffness and dashpot distributions at the contact nominal interface, allowing to upscale at the macroscale the influence of dominant contact mechanisms occurring at the roughness scale. In this context and as reviewed by Baltazar and coworkers [1], starting from fundamental results of classic contact theories and accounting for main microgeometric features at the contact interface, several theoretical and numerical models have been proposed in the specialized literature (namely, spring-like models), aiming to consistently derive some equivalent stiffnesses.

In this work a novel spring-like theoretical model for no-sliding contact under a closure pressure is proposed. Incremental normal and tangential equivalent stiffnesses at the nominal contact interface are derived, by assuming contact microgeometry be described by isolated *internal* cracks [2] occurring in a thin interphase region (see Figure1).

In detail, effective mechanical properties at the contact zone are consistently derived following the imperfect interface approach recently adopted by Lebon and coworkers [3, 4], by coupling a homogenization approach for microcracked media based on the non-interacting approximation [5, 6] and the matched asymptotic expansion method, employed, among others, by Lebon and Rizzoni [7].

An analytical description of evolving contact and no-contact areas with respect to the closure pressure is also provided, resulting consistent with theoretical Hertz-based asymptotic predictions and in good agreement with available numerical estimates. Proposed model has been successfully validated through comparisons with some theoretical and experimental results available in literature, as well as with other well-established

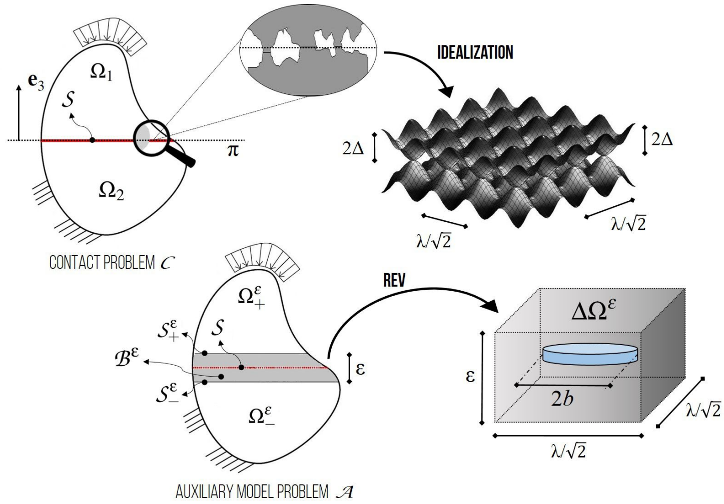

Figure 1: Contact problem $\mathcal{C}$ and auxiliary model problem $\mathcal{A}$.

modeling approaches [8]. Finally, the influence of main model parameters is addressed, proving also the model capability to catch the experimentally-observed dependence of the tangent-to-normal contact stiffness ratio on the closure pressure.

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A finite element study of the surface roughness in elasto-plastic shrink fitted joint

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Summary: The paper presents some remarks on factors affecting the characteristics of shrink–fit assemblies in presence of surface roughness and micro–slip. The resulting factors influencing the contact zone will be explained in terms of the deformation of asperities which are always present, even on the finest machined surfaces.

Interface model

When these surfaces are loaded together, contact is made at the asperity summits. It is generally observed that the better surface finish the strength of the assembly is greater [1]. Only few studies have been done to take roughness into account in designs of interference–fit assemblies [2, 3]. A phenomenological description of the frictional phenomena is based here on a similarity of friction and elasto-plastic behaviour. For metallic bodies a non-associated slip law is adopted in which $f \neq g$.

The normal stiffness per unit area is taken from one single asperity consideration by finte element method and then multiplied on all contacting asperities using the second order probability density function. The surfaces of the samples were subjected to three kinds of mechanical processes: fine (FSB) and coarse sandblasting (CSB) and electrical discharge machining (EDM) [4].

Numerical results

An assembly under examination is cylindrical joint which consists of two main elements: a shaft (length 60 mm, diameter 20.02 mm), hub (length 20 mm, inside diameter 20 mm, outside diameter 30 mm). Both the shaft and hub are made from the same material (steel). The following material constants have been adopted for calculations: $E = 2 \cdot 10^5$ N/mm^2, $\nu = 0.3$, thermal expansion coefficient $\alpha = 2 \cdot 10^{-4}$ 1/°C. Calculations have been carried out for elastic as well for elasto–plastic material properties of the hub (see Fig. 1 and Fig. 2).

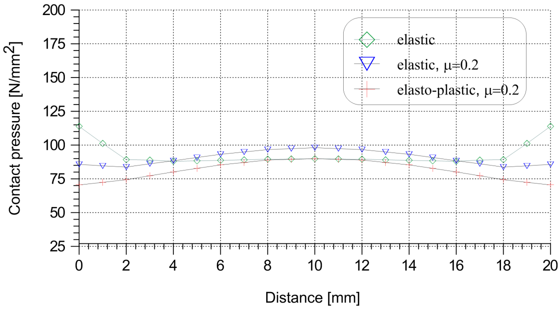

Figure 1: The contact pressure for EDM surface: interference tightening $\delta = 0.02$ [mm], $\sigma_Y = 200$ [MPa].

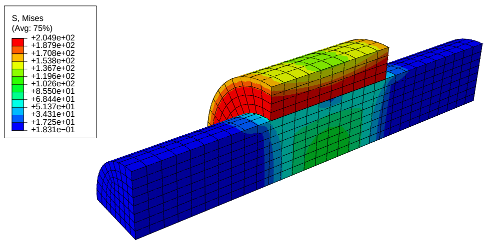

Figure 2: The von Mises effective stresses for EDM surface: interference tightening $\delta = 0.02$ [mm], $\mu = 0.2$, $\sigma_Y = 200$ [MPa].

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Multi-scale asymptotic modelling of a cluster of adhesive micro-contacts

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A multi-scale asymptotic modelling approach is applied to the problem of JKR type adhesive frictionless contact for a cluster of micro-contacts. The asymptotic solution is obtained by the method of matched asymptotic expansions and used in the analysis of the contact splitting principle.

In recent years, much attention has been given to adhesion of biological systems [1–3]. For the so-called dry adhesive systems, like those of spiders and lizards, the explanation of the adhesion mechanism was given [4, 5] in the framework of the classical JKR (JohnsonKendallRoberts) model [6], originally derived for single spherical contacts. In particular, the principle of contact splitting [5] was introduced stating that the adhesion force for a given apparent contact area increases as the total contact is split up into ever-finer contact elements. This result, which follows from the dimensional reasons inherent in the JKR analysis, was obtained under the simplifying assumption of non-interacting micro-contacts. In the present research we re-examined the problem of adhesive contact for a system (commonly named cluster) of micro-contacts using the multi-scale asymptotic modelling approach [7]. It is shown that the interaction between the micro-contacts reduces the predicted pull-off force.

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Contact assisted auxetic microstructures

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Summary: Topology optimization for compliant mechanisms can be used for the design of novel mechanical microstructures having auxetic behaviour. The auxetic behaviour of the resulting microstructure is influenced by unilateral contact between constituents, large displacements and elastoplasticity. Verification is done using numerical homogenization.

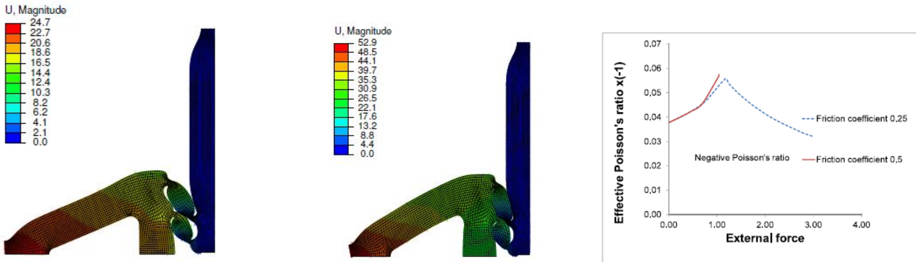
Introduction

The general scheme of topology optimization and design for auxetic materials will be initially presented in this article. Then, a final assessment of the material distribution given by topology optimization, with a non-linear finite element code, will be given. Therefore, the influence of contact and friction between the constituents, will be studied. It will be shown also that geometric non-linearity and elastoplasticity have a significant influence on the auxetic behaviour.
The possibilities and limitations of topology optimization for the design of compliant mechanisms and auxetic microstructures can be found among others in [1, 2]. In addition, the ability of verifying results obtained by topology optimization by using numerical homogenization is shown in [3]. First results related to the present work can be found in [4].

Topology optimization and post-processing by using contact mechanics

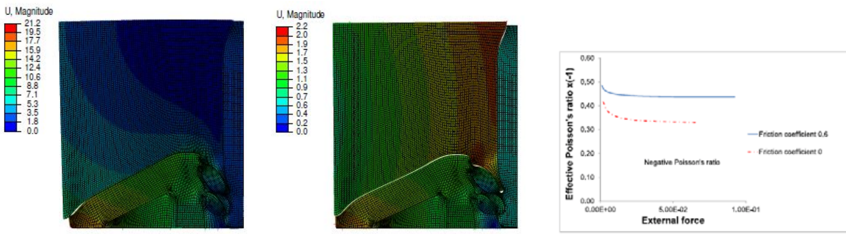
In this work, first, a robust hybrid algorithm based on evolutionary algorithms and local search steps is used as an evaluation operator. Several random material distributions are generated and evaluated using topology optimization. Then, the final material distributions are developing according to each evolutionary algorithm rules and strategies which are evaluated again using topology optimization. The algorithm stops until termination criteria are met. The results are four different auxetic microstructures based on a 120 X 120 RVE composed of 2-D plane stress elements. The case with the highest Poisson's ratio is obtained and then a structural optimization refinement and simplification of the overall procedure is implemented by means of numerical homogenization techniques. Variations to this scheme using multiple criteria are also possible.
The given auxetic geometry, was used for the development of respective finite element models. Within these models, both the hard and the soft matrix material (composite) or only the hard material (microstructure) are considered. Geometric, as well as material non-linearities are incorporated to the model, combined with the non-linear unilateral contact between the constituent materials of the microstructure, in order to identify how the auxetic behaviour of the material is influenced by these non-linearities, which actually appear to a real material. The given model enriched with the appropriate contact interfaces, different loads incorporated and elastoplasticity were activated. The non-linear finite element analysis depicted the behaviour of the microstructure and for each simulation and loading level the effective Poisson's ratio was calculated.

Useful conclusions are derived from these finite element analyses. The implementation of contact conditions with several values for the friction coefficient between the constituent materials changes the deformation mode of the structure, since opening and/or sliding appear in the interfaces. The auxetic behaviour is influenced also in this case, when contact between both materials or self-contact in the hard material occurs (Figures 1, 2).
Furthermore, geometric non-linearity and plasticity influence the development of the effective Poisson's ratio (and the auxetic behaviour). Appearance and expansion of plastic regions in the material may cancel the auxetic behaviour. When a tie constraint is applied between the constituents, the auxetic behaviour progressively seems to disappear and for opposing forces a symmetry at the material behaviour is observed.



(a) (b) (c)

Figure 1. Self contact of the hard material (a) with friction coefficient 0.5, (b) 0.25, (c) diagrams



(a) (b) (c)

Figure 2. Opening-sliding and deformation of both materials (a) friction coefficient 0.6, (b) 0, (c) diagrams

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Flattening of loaded rough surfaces: normal contact versus sliding contact

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Summary: The problem of contact of deformable rough surface with rigid flat counterpart was investigated theoretically and experimentally. In the special experimental setup two modes of rough surface flattening were performed: normal compression and sliding (tangential load) in presence of normal compression. The resulting deformation of roughness zone has been analyzed.

Introduction

Flattening of rough surfaces due to contact load is a phenomena observed in different technical applications. Plastic surface deformation of metal parts is a process that lead to the smoothing of rough surfaces. In the case of purely normal load the problem was exhaustively investigated by many researchers [1-3]. When a tangential load with sliding is present the problem is more complicated. The number of detailed studies concerning the problem is not so large [4,5,6]. The analysis of such problem is important in manufacturing engineering (e.g. metal forming) where the surface quality is an important factor. A plastic smoothing tool should have high rigidity, precision, and wear resistance; its surface quality should be better (the roughness lower) than that of the part under machining. In this process the friction coefficient is also an important factor [7].

In the paper the deformation of roughness zone in contact with rigid flat surface is investigated. The deformed surface was next examined using scanning profilometry and the evolution of the real contact area (RCA) and surface parameters were specified.

Experiment and results

Samples were made of two kinds of steels (carbon steel C45 and s235) with different stress – strain characteristic and then were subjected to mechanical surface treatment (shot peening), that generates surfaces topography having similar characteristics on each steel. The experiment was carried out using a modified device described in [3] which enables precise measurement of the approach as a function of the contact pressure. Two kinds of experiment were considered: in the first the normal load was applied and in the second both normal load and tangential load resulting from sliding motion of the rigid plane were present. The second experiment was performed in two stages: first the normal load is applied and next maintained during sliding of contacted surfaces. In the experiment the approach of surfaces as a function of normal load was measured.

The roughness parameters have been measured on a scanning profilometer before and after loading. The changes of the height parameters and RCA are presented in the Fig.1 and Fig. 2 respectively. The procedure of RCA estimation based on the profilometric measurement of the surface is described in [3]. The results presented in Fig.2 show that there is not difference between RCA for investigated steels after loading with sliding but after pure normal loading this difference is evident. It has been observed that when the sliding take place, one can apply much lower normal load to achieve a similar deformation of the roughness zone as in the case of purely normal load (Fig.3).

The simple model based on statistical approach proposed by Greenwood Wiliamson in 1966 and finite element solution has been applied to predict the relations load-penetration depth and load-real contact area. The model prediction was compared with experimental results.

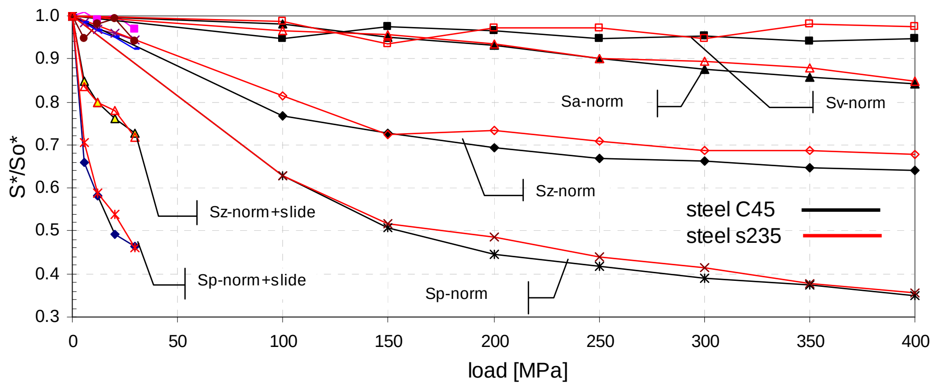

Figure 1: Height parameters vs. normal loading *(norm)* and loading with sliding (*norm+slide*).

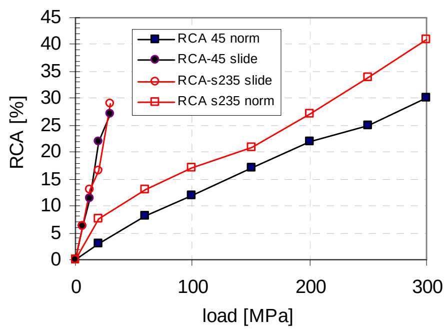

Figure 2: Real contact area (RCA) vs. normal loading and loading with sliding

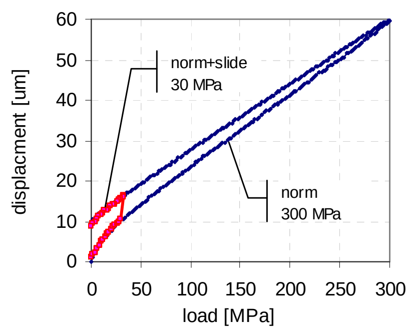

Figure 3: Displacement vs. normal loading and loading with sliding

Conclusions

The presence of tangential load (sliding motion) in the contact of rough surfaces loaded with normal pressure leads to a serious change of its plastic compliance. Applying a tangential force, one can diminish approximately ten time the normal load to achieve the same deformation of the roughness zone as in the case of purely normal load (Fig. 1,3). This effect has been confirmed both experimentally and theoretically.

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On the homogenization of rubber-rough-surface-contact using an adaptive stochastic collocation scheme

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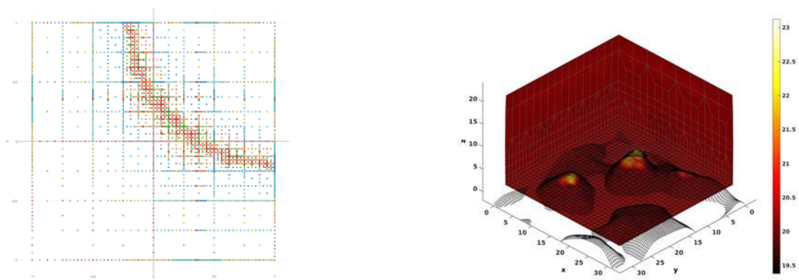
Summary: A stochastic sampling method for the contact of tire tread rubber with rough road surfaces accounting for real-world operational conditions and material models will be presented. These investigations aim for more realistic computations in tire mechanics using statistically homogenized contact laws derived from these investigations.

Introduction

The performance of car tires is essentially influenced by the contact behavior of tread rubber with the rough road surface. Goals like traction and wet grip, rolling resistance, wear, and tire noise excitation are often seen as leading to contradictory design choices. An improved understanding of tire tread rubber contact with rough road surfaces under realistic operational conditions will assist in optimizing this system.
On modern asphalt roads with rough texture, the tread rubber is only in contact with a few of the highest asperities, as depicted in figure 1(b). Local penetrations of about 30% strain at frequencies of up to 100 Hz arise under these conditions. Thus, kinematic as well as material nonlinearities have to be considered.
In addition, a contact patch hits the random surface texture randomly, i.e. in the next rotational cycle, the same patch will be penetrated by other asperities at different positions.
Goal of this research project is to provide homogenized, stochastic reaction forces of a simple rubber tread block in contact with the rough surface.



(a) (b)

Figure 1: Stochastic simulation: (a) adaptive stochastic collocation scheme in the parameter space, (b) temperature distribution due to dissipative heating caused by rough surface contact.

The computational approach

Rough surface generation and tread block modeling

Rough surface textures are reconstructed from measurements of real roads. The data is processed via DFT and band-pass-filtered in order to achieve a more robust contact computation.

Simple tread block geometries are assumed as a first approximation, discretized with gradual refinement towards the contact surface. Detailed thermo-viscoelastic material models for rubber materials at finite strains are employed in these computations [1].

A staggered Dirichlet-Neumann contact scheme is applied to obtain an initial guess for contact algorithms based on constrained optimization, leading to more robust computations.

Sampling techniques

A Multilevel Adaptive Sparse Grid Collocation (MLASGC) scheme (see figure 1(a)) was developed recently [2] and will be applied in obtaining converged stochastic response surfaces at a fraction of the computational cost required by non-adaptive single-level sampling schemes.

Computational Studies

Two different studies on the stochastic contact homogenization will be presented, the first one with emphasis on rolling loss simulation and the second one with emphasis on rolling noise simulation. In both studies, the tire structure is subjected to high-frequency excitation caused by rough surface contact.

In the first application, a coupled thermo-mechanical simulation of the meso-scale contact is investigated. From the stochastic approach, a homogenized contact law for the contact pressure and the viscous heat generation within the bulk material is derived, which can be applied immediately to macroscopic rolling tire analysis.

From the second investigation, a stochastic transient excitation is derived, which can be transformed to a random excitation function for the contact nodes of a stationary rolling tire structure. This excitation function can be used as an input to a modal superposition approach [3], yielding a detailed operational tire vibration analysis.

Summary

In contrast to many previous investigations on rough surface contact which rely on linearity assumptions with regard to the material behavior and stationarity conditions, the presented computational approach enables far more realistic studies of the contact of tire tread rubber with rough road surfaces under true operational conditions when coupled with macroscopic rolling tire simulations [4]. First results demonstrate the computability and motivate future research in this direction.

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Validation of a multiscale FEM approach for elastomer friction on rough surfaces

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Summary: A multiscale FEM approach for elastomer friction on rough surfaces is proposed and validation studies evaluating experimental data for different surfaces, materials and global conditions are presented.

Understanding the frictional behaviour of elastomers on rough surfaces is of high practical importance in many industrial applications. For example the traction of a tire is directly linked to the material properties of the considered elastomer and the surface conditions of the road track, see [1]. One goal of our studies is to gain a deeper understanding of the underlying contact physics at all length scales. Another aim is to determine a macroscopic coefficient of friction for varying global conditions, material and surface properties and to validate the results with experimental data.

For predicting the coefficient of friction certain physical effects like hysteresis, adhesion or flash temperature effects have to be taken into account. One of the main aspects of elastomer friction on rough surfaces is the internal energy dissipation due to cyclic loading and unloading, called hysteresis. This effect causes also the heating of the elastomer sample. The micro roughness of the surface contributes mainly to the hysteresis component of rubber friction, see [2] or [3]. The adhesion phenomenon is caused primarily by rubber molecules undergoing bonding and debonding cycles at the rough counter surface at nanometer length scale. To capture all details and information down to micro scale at acceptable computational costs it becomes necessary to incorporate all coupled physical aspects into a multiscale framework.

In this study a multiscale finite element approach for elastomer friction on rough road surfaces is presented. The rough surface is decomposed in a number of scales and homogenized friction laws are passed from microscopic representative computations up to the largest scale, see [4]. For modelling the coupled effect of hysteresis and flash temperature in the multiscale framework a finite linear thermomechanical viscoelastic material model containing a series of maxwell elements is used for the elastomer. The adhesive part of rubber friction is modelled by a phenomenological friction law incorporated at the macroscopic scale since complex interactions at a submicron scale are not considered by the proposed FEM approach. Details of the developed multiscale framework and validation studies for different physical effects and global conditions are presented.

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Contact of viscoelastic bodies with periodically wavy surfaces

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Summary: The present work aims to elaborate a computational approach based on the boundary element method for solving the frictionless contact problem of two viscoelastic bodies with periodically wavy surfaces. The proposed numerical approach turns out to be particularly simple and efficient, because only the nominal contact area relative to one period needs being meshed. Numerical examples are provided to test and illustrate the simplicity and efficiency of the proposed computational approach.

The contact between viscoelastic and rough surfaces is a problem often met in a variety of situations. This problem is much more complex than the elastic counterpart, because of its dependence on time. The present work has the primary purpose of elaborating numerical approach to analyzing the contact between a rigid periodically wavy surface and the smooth surface of a viscoelastic half-space. The pressure distribution and the contact area are investigated by using the matrix inverse method [1,2]. By exploiting the corresponding periodical condition, the contact problem can be formulated as follows

$$u(x,y;t) = \int_{t_0}^{t} E(t-\tau)\frac{d}{d\tau}\left[\iint_{\Sigma} A(x,y;\xi,\eta;\tau)p(\xi,\eta;\tau)d\xi d\eta\right]d\tau, \tag{1}$$

where $u(x,y;t)$ is the normal displacement of the point (x,y) of the viscoelastic surface at instant t, $p(x,y;t)$ is the contact pressure distribution, Σ is the reference periodic cell of the viscoelastic surface, $E(t-\tau)$ is the creep compliance function and t_0 is the initial instant. The function $A(x,y;\xi,\eta;\tau)$ is given by

$$A(x,y;\xi,\eta;\tau) = \frac{1-\nu(\tau)}{\pi}\sum_{k=1}^{N_p}\frac{1}{\sqrt{(x-\xi-x_k)^2+(y-\eta-y_k)^2}}, \tag{2}$$

where $\nu(\tau)$ is the Poisson's ratio of the viscoelastic half-space at the instant τ, (x_k, y_k) are the coordinates of the center of the period k and N_p is the number of period considered. The number of periods N_p is theoretically infinite but it is sufficient to choose a finite value for numerical calculations [3]. Then only one cell of the viscoelastic surface is needed to be meshed. As an example, we deal with the indentation of a viscoelastic half-space by a rigid surface with a sinusoidal profile in the x and y directions. The particular case where the Poisson's ratio is time-independent is numerically studied. In addition, the behavior of the viscoelastic half-space is described by the rheological model of Zener; the creep compliance function for Young's modulus is given by

$$E(t) = \frac{1}{E_\infty} - \frac{E_0 - E_\infty}{E_0 E_\infty}e^{-(E_0/E_\infty)t/\tau}, \tag{3}$$

where E_0, E_∞ and τ are material parameters of the viscoelastic half-space. The initial gap between the sinusoidal surface and a point (x,y) of the viscoelastic surface is given by

$$g(x, y) = \Delta\left[2 - \cos\left(\frac{2\pi x}{\lambda}\right) - \cos\left(\frac{2\pi y}{\lambda}\right)\right], \quad (4)$$

where λ is the wavelength in the x and y directions and Δ is a parameter controlling the amplitude of the sinusoidal roughness.

The numerical values $E_\infty = 210$ GPa, $E_0 = 2E_\infty$, $\nu = 0.3$, $\tau = 0.001$ s, $N_p = 51$, $\Delta = 0.01$ mm, $\lambda = 10/51$ mm are adopted for the numerical simulation. The prescribed normal displacement of the rigid indenter in the viscoelastic half-space is chosen to be $\delta = 0.1$ mm. The evolution of the total contact force with time is shown in Fig. 1. Note that it tends to the elastic one after some time.

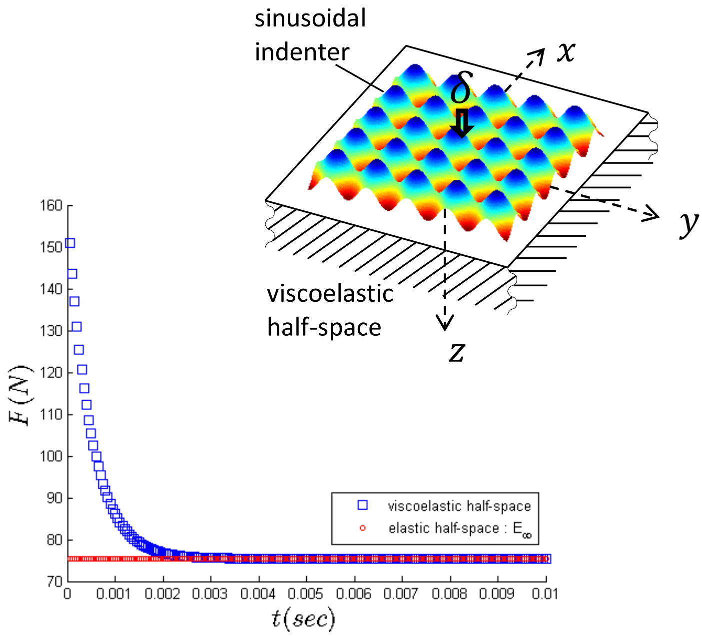

Figure 1: Total contact forces vs time in the case of the indentation of a viscoelastic half-space (blue squares) and an elastic half-space with Young's modulus E_∞ (red circles) by a wavy rigid surface.

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Contact of multi-level periodic system of indenters with homogeneous and two-layered elastic half-space

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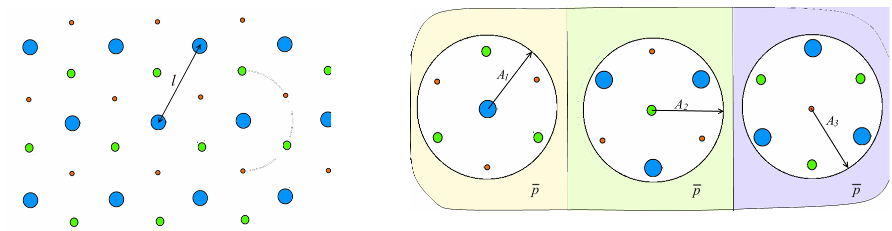
A periodic system of indenters with several height levels and different size for each level is considered in contact with homogeneous or layered elastic half-space. Numerical and analytical methods are developed to find contact pressures for each level taking into account mutual effect. The results for relatively hard and soft coatings are analyzes and compared with the results for homogeneous half-space.

Introduction

Roughness in contact problems is often modeled by periodic system of indenters to analyze mutual effect for different model geometry and to study real contact area as a function of average load applied to the period [1]. The method of the periodic contact problem solution for coated bodies was developed in [2]. In this paper multi-level periodic contact problem is considered for better simulation of real roughness. Each level has its geometry of indenters. The counter-body is homogeneous elastic half-space with or without elastic coating.



(a) (b)

Figure 1: (a) Location of indenters in 3-level model, (b) scheme of calculations.

Problem formulation and the method of solution

Let's consider a periodic system of indenters (figure 1,a). The contact surfaces are described by smooth functions $z = f_m(r) + h_m$, where h_m (m=1..k) determines the m-level height. The contact conditions are:

$$w(\bar{r}) = f_m(\bar{r} - \bar{r}_i) - \delta, \quad \bar{r} \in \omega_{im},$$
$$\sigma_z = 0, \quad \bar{r} \notin \omega_{im}, \quad m = 1..k,\ i = 1,2,...,\infty, \qquad (1)$$
$$\tau_{rz} = 0, \quad \tau_{\theta z} = 0, \quad 0 \le |\bar{r}| < \infty$$

Here ω_{im} is a contact spot for m level, w, σ_z, τ_{rz}, $\tau_{\theta z}$ are displacements and stresses. The coating-substrate conditions for two-layered half-space correspond to the case of perfect adhesion. Localization method [1] is used to solve the contact problem (figure 1,b). Real contacts are considered only inside a circle with an indenter as a centre. The circle radius is

$$A_m^2 = \frac{1}{\pi}\left(\sum_{j=1}^{k}\frac{k_{jm}}{\bar{N}_j} + \frac{1}{\bar{N}_m}\right), \quad (2)$$

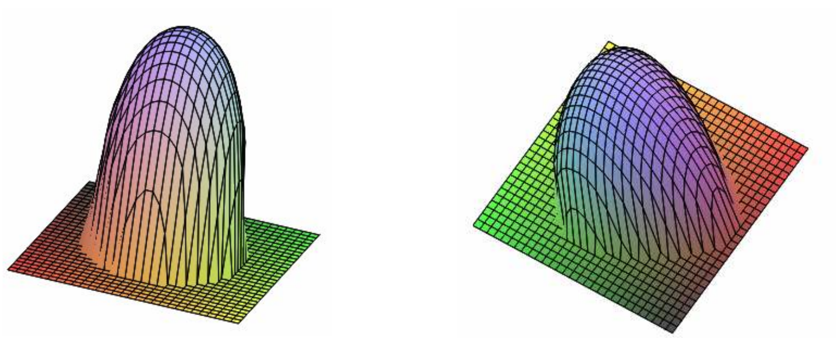
where $\bar{N}_j$ and $\bar{N}_m$ - the density of indenters of *j* and *m* levels, k_{jm} - the number of indenters of *j* level inside the circle. Outside the nominal contact pressure $\bar{p}$ is considered:

$$\bar{p} = \sum_{j=1}^{k}\bar{N}_j \iint_{\omega_j} p_j(r,\theta) r dr d\theta \quad (3)$$

The contact problem is solved step by step from low loads, which provides only the contact of the first level. For the case the method of contact problem solution for coated bodies [2], which is based on Hankel and Fourier integral transforms, boundary elements method and iterations, can be used. The initiation of the next level contact requires double iteration procedure to take into account mutual effect.

Results and discussion

The example of contact pressure distributions under an indenter of the first level is presented in figure 3 for the case of relatively hard coating. When only first level is in contact, the pressure distribution has axial symmetry; but when two levels are in contact, non-symmetry arises. The mutual effect is not so strong for relatively soft coatings. It also depends on the coating thickness, for relatively large thickness the results tend to contact pressure distribution obtained for the homogeneous body.
Relative real contact area is determined here as a ratio of real contact inside the circle with radius A_m to the square of the circle. It is analyzed as a function of $\bar{p}$ for relatively hard and soft coatings, and for homogeneous half-space.



(a) (b)

Figure 2: Contact pressure distributions under an indenter of the first level, (a) only first level is in contact, (b) two levels are in contact (3-level model).

The work was supported by Russian Science Foundation (No. 14-19-01033).

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Elasto-Plastic Rolling Contact Problems with Nonhomogeneous Materials

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Summary: The paper deals with the numerical solution of thermo-elasto-plastic wheel-rail contact problems including Coulomb friction, frictional heat generation and heat transport across the contact surface. The rail surface is assumed to be covered with a coating layer made from the functionally graded material. The mechanical and thermal properties of this layer are changing with its depth. This layer and the rail can undergo elastoplastic deformations. The contact phenomenon is governed by the coupled system of elastoplastic and heat conductive equations. Finite difference method and finite element methods are used as to discretize the original problem. The discretized contact problem is solved numerically using the semi-smooth Newton method. Numerical examples are provided and discussed.

Introduction

This paper deals with the numerical solution of the wheel-rail contact problems including friction and frictional heat generation. The contact of a rigid wheel with an rail lying on a rigid foundation is considered. The friction between the bodies is governed by the Coulomb law [2–4]. Numerous laboratory or numerical experiments [2] indicate that the use of a coating functionally graded material attached to the conventional steel body reduce the magnitude of residual or thermal stresses and rolling contact fatigue. Functionally graded materials are multiphase composites mainly composed of a ceramic and a metal. They exploit the heat, oxidation and corrosion resistance typical of ceramics, and the strength, ductility and toughness typical of metals.

In this paper we solve numerically the wheel-rail rolling contact problem with friction assuming plastically graded model of the coating layer rather than elastic as in [2]. The time-dependent model of this rolling contact problem is introduced. The elastic and plastic responses of the graded layer are approximated, respectively, by Hooke's law and the von Mises yield criterion with isotropic power law hardening [1, 4]. Finite difference and finite element methods are used as discretization methods. The discretized problem is formulated in terms on nonlinear complementary functions and solved using semi-smooth Newton method [3]. The distribution of stresses including the normal and tangent contact stresses as well as the distribution of the temperature are numerically calculated. The provided results are discussed.

Thermo-Elstoplastic Contact Problem

Consider deformations of a two-dimensional strip lying on a rigid foundation. A wheel rolls along the upper surface of the strip. The axis of the wheel is moving along a straight line at a constant altitude and the wheel is pressed in the elastoplastic strip.

It is assumed, that the head and tail ends of the strip are clamped, i.e., we assume that the lenght of the strip is much bigger than the radius of the wheel. Moreover it is assumed, that there is no mass forces in the strip. The strip consists from two layers. The mechanical and thermal properties of the surface layer are assumed vary throughout the material in the vertical direction according to the power law. The Poisson's ratio is assumed constant. Conventional model of isotropic plasticity is assumed. The yield function depends on effective Mises stress, the yield stress on the upper surface and the isotropic hardening function. Since the rolling contact problem involves thermo-mechanical coupling it requires to develop separate algorithms for the thermal and elastoplastic analysis. The heat flow is governed in both layers by the heat conduction equations. The heat generated by the plastic deformation is assumed to be suitably small. The boundary conditions associated with the heat conduction are due to the contact between layers and the surrounding environment. The contact between the layers generates heat flow due to friction, and the frictional heat flux is proportional to the friction coefficient, the contact pressure and the sliding velocity. For details see [2]. A standard contact formulation is used imposing the continuity of normal displacement across the contact interfaces if the contact condition is satisfied.

Numerical implementation

The finite difference and finite element methods are used as the disretization methods. In the numerical procedure the thermal and elastoplsatic contact problems are solved sequentially in time. For the contact pressure calculated in the previous time step the heat flux and the temperature are evaluated for the next time step. If the temperature increament is less than the prescribed value the algorithm is stopped. Otherwise the contact pressure for the next time step is evaluated. Here we use semi-smooth Newton method [3] to solve the elastoplastic contact problem. This approach consists in reformulating the complementarity conditions as a set of semi-smooth equations.

Conclusions

The obtained numerical results indicate that the contact patches are characterized by longer zones and lower stress intensity than in the elastic case. Moreover the graded layer can reduce the values of the normal contact stress and the maximal temperature in the contact zone. The obtained stress and temperature distributions are dependent on the nonhomogenity index. For higher values of it one can obtain the higher differences in the maximal contact stress and temperature in comparison to the homogeneous material case.

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Anisotropic friction and wear rules with account for anisotropy evolution

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Summary: In the paper the effect of anisotropy and of variation of surface topography on friction and wear process has been investigated theoretically and experimentally. The evolution of roughness and friction coefficient during wear process is analysed experimentally. The model predicting evolution of wear rate and friction coefficient has been proposed.

Introduction

The anisotropic friction response in sliding along contact surfaces is usually associated with distribution of surface asperities in the form of oriented striation patterns induced by machining or polishing processes. Such response is also observed in fiber-reinforced composite materials where the significant effect of fiber orientation on frictional behaviour and wear has been observed, cf. [1]. The friction anisotropy occurs also when sliding proceeds along the mono-crystalline surface.

The present study is related to analysis of coupled friction and wear process in sliding along the rough surface with an anisotropic asperity pattern, characterized by mutually orthogonal striations. Due to wear process the initial anisotropic response evolves with the asperity distribution tending to a steady-state pattern, usually corresponding to the isotropic response. The friction response is also affected and reaches its steady state.

Orthotropic friction model

Following the previous papers by Michalowski and Mróz [2], Mróz and Stupkiewicz [3] consider an orthotropic surface roughness with the parallel layout of asperities, Fig.1

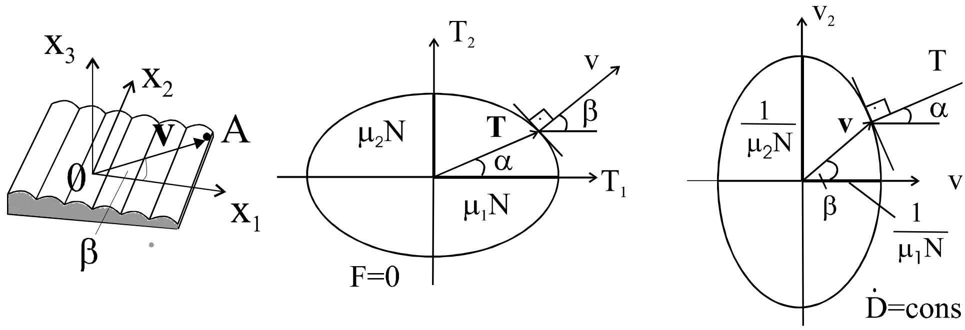

Fig.1 Loci of limit friction condition *F=0* and of constant dissipation rate, $\dot{D}$*=const* for an orthotropic roughness profile

The limit friction condition is assumed in the form

$$F(T_1,T_2)=\sqrt{\left(\frac{T_1}{\mu_1}\right)^2+\left(\frac{T_2}{\mu_2}\right)^2}-N\le 0 \tag{1}$$

where T_1, T_2 are the tangential contact tractions parallel to the orthotropy axes 1, 2. The normal contact traction N is assumed positive, $N>0$, for the case of compression. The non-associated sliding rule was derived in [2, 3] but for the simplicity the associated rule is assumed, thus

$$v_1 = \dot{\lambda}\frac{\partial f}{\partial T_1} = \dot{\lambda}\frac{T_1}{\mu_1^{\ 2}}, \qquad v_2 = \dot{\lambda}\frac{\partial f}{\partial T_2} = \dot{\lambda}\frac{T_2}{\mu_2^{\ 2}} \tag{2}$$

In view of (1), (2), there is

$$\dot{\lambda} = \dot{D} = N\sqrt{(\mu_1 v_1)^2 + (\mu_2 v_2)^2} == T_1 v_1 + T_2 v_2 \tag{3}$$

and the dissipation rate $\dot{D}$ is expressed by the multiplier $\dot{\lambda}$. Here μ_1 and μ_2 are the principal values of the friction tensor **M** with principal directions coinciding with the orthotropy axes 1, 2. Now we have the inverse relations

$$T = \frac{\partial D}{\partial v_1} = N\frac{\mu_1^2 v_1}{\sqrt{(\mu_1 v_1)^2 + (\mu_2 v_2)^2}}, \qquad T_2 = N\frac{\mu_2^2 v_2}{\sqrt{(\mu_1 v_1)^2 + (\mu_2 v_2)^2}} \tag{4}$$

and

$$\frac{T_2}{T_1} = \tan\alpha = \frac{\mu_2^2}{\mu_1^2}\frac{v_2}{v_1} = \frac{\mu_2^2}{\mu_1^2}\tan\beta \tag{5}$$

where α and β denote the angles of the tangential force ***T*** and the sliding velocity vector ***v*** to the orthotropy axis 1.

Wear rule accounting for the asperity evolution

Assume the wear rate to be proportional to the friction dissipation rate, thus

$$\dot{w}_n = k_w\dot{D} = k_w N\sqrt{(\mu_1 v_1)^2 + (\mu_2 v_2)^2} = k_w N v\sqrt{(\mu_1\cos\beta)^2 + (\mu_1\sin\beta)^2} \tag{6}$$

where k_w is the wear factor and *v* is the sliding vector modulus. Such wear rule was used in [4] in numerical analysis. However, due to wear process, the friction moduli μ_1 and μ_2 are assumed to vary, thus

$$\mu_1 = \mu_{1c} - (\mu_{1c} - \mu_{10})e^{-\kappa\lambda}, \qquad \mu_2 = \mu_{2c} - (\mu_{2c} - \mu_{20})e^{-\kappa\lambda} \tag{7}$$

where μ_{1c}, μ_{2c} are the asymptotic principal values of the friction tensor **M** and μ_{10}, μ_{20} are the initial values, *k* denotes the material parameter. Relations (7) can be expressed as

$$\dot{\mathbf{M}} = \kappa(\mathbf{M}_c - \mathbf{M})\dot{\lambda}, \qquad \mathbf{M} = \mathbf{M}_c - (\mathbf{M}_c - \mathbf{M}_0)e^{-\kappa\lambda} \tag{8}$$

where $\lambda = \int\dot{\lambda}dt = \int\dot{D}dt$ is the total frictional dissipation. In particular, it can be assumed that $\mu_{1c} = \mu_{2c} = \mu_c$ and then the asperity pattern tends to *isotropic* distribution in the wear process.

Experimental verification

The wear tests have been executed by means of a ball-on-disk wear tester by applying the reciprocal ball motion (stroke length 3mm, frequency of ball oscillation ω = 4.461 s^{-1}) in unlubricated wear conditions against flat specimen., (cf. tests for isotropic friction [5]). The sapphire (AL_2O_3) ball of diameter *D=6mm* was used. The wear of grinded surface characterised by long grooves was tested. Three directions of ball motion with respect to grooves generated in surface finishing process have been tested: longitudinal, oblique and transverse directions. Different values of normal load were applied: 0.05N, 0.1N and 0.5N. The evolution of friction coefficient in the initial stage of wear is presented in Fig. 2 for longitudinal and transverse sliding directions.

One can observe that initially, the friction coefficient corresponding to the transverse direction is greater than that corresponding to the longitudinal direction. In this stage of test the the initial state of surface roughness is decisive for value of μ.

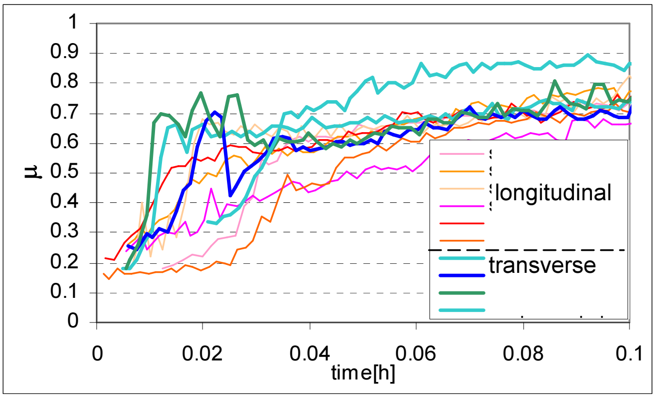

Fig. 2 Evolution of friction coefficient for 0.05N normal load

In the next 5-7 minutes of test duration the sub-roughness is generated at the worn summits, Fig. 3, and μ gradually increases. At the end of this period a new form of roughness and consequently new contact conditions occur. Then a new value of friction coefficient is established that is common for both sliding directions. Thus, the developed asperity pattern assures the isotropic friction and wear growth.

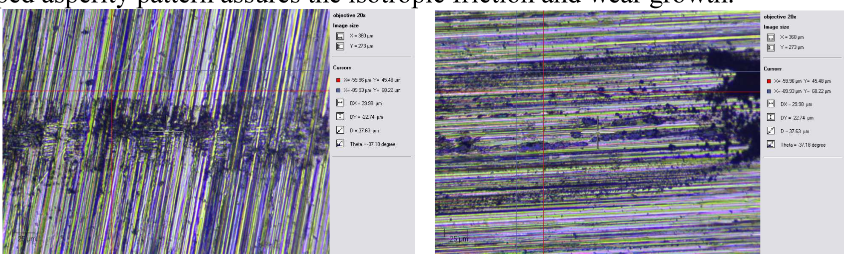

Fig. 3 Worn summits of the initial roughness in a first stage of friction for transverse and longitudinal sliding direction

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Integral equation with variations of domain : applications to wear contact

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The friction and wear phenomena appear due to contact and relative motion between two solids. The evolution of contact depends on loading conditions and mechanical behaviours of the solids.The geometry of the contact surface evolves with wear which is essentially characterized by matter loss. We investigate the influence of this evolution on the contact condition using integral equations describing the problem of elasticity on an half plane.

Introduction

The wear phenomena are induced by contact and relative motion between two solids. They depend on the loading conditions and on the materials properties; they are essentially characterized by a matter loss. Particles are detached from the solids in contact and a complex medium takes place between the two solids forming a thin layer. The contact conditions, the geometry of the contact surface evolve simultaneously. Such can be described in a thermodynamical approach of running discontinuities [1]. The wear phenomena are related to the dissipation process associated to the loss of matter which is defined in terms of an energy release rate G distributed along the surface of contact. The wear process is defined by a kinetic relation $\phi = \mathcal{F}(G)$. In this description ϕ is the normal velocity of the boundary of the solid in contact.

A particular example

Consider a rigid punch moving on a elastic half-elastic plane, and a layer composed of a viscous fluid with particles in suspension. The global behaviour of the fluid is defined by a bulk modulus κ and a viscous modulus η which are function of the concentration of particles. This particular case of contact has been studied using integral equations [2]. The punch is moving with a constant velocity V. The half plane has an elastic behaviour, the displacement u, v on the boundary is given by solving the Galin's equations [3]:

$$C_1 u_(x,x)(x) = C_2 \sigma_{yy} + V_p \frac{1}{\pi} \int_{-a}^{+a} \frac{\sigma_{xy}(s)}{x-s} ds, \quad (1)$$

$$C_1 u_(y,x)(x) = -C_2 \sigma_{yx} + V_p \frac{1}{\pi} \int_{-a}^{+a} \frac{\sigma_{yy}(s)}{x-s} ds, \quad (2)$$

where $C_1 = \dfrac{E}{2(1-\nu^2)}, C_2 = \dfrac{1-2\nu}{2(1-\nu)}$, E is the Young modulus, ν is the Poisson ratio, $V_p f$ is the principal value of f in the sense of Cauchy. The evolution of the surface $y = 0$ is given by a constitutive law, $\phi = \lambda < \sigma_{yy}^2 - \sigma_c^2 >$. The relation between the displacement of the punch and the layer is given by $\delta - u_y^s(x)$ and the behaviour of the layer is reduced to the relation for the shear and the normal force

$$\sigma_{xy} = m(c)(\dot{u}_x^1 - \dot{u}_x) \quad \sigma_{yy} = \kappa(c)(u_y^1 - u_y) \quad (3)$$

The velocity is given by the steady state condition. The conservation of the mass of particles gives a relation between the concentration c and the velocity ϕ. The moduli $m(c)$ and $\kappa(c)$ are given by classical equations of mixture.

The perturbation of the boundary

The evolution of the Galin's equation with respect to the motion of surface $y = 0$ is obtained by a perturbation $y = \epsilon\eta(x)$ of the boundary integral equation in elasticity taking account of the Green function under plane strain condition.

$$u = \int_{-a}^{a} G(x-s, z-\epsilon\eta(s)).T(s)J(s)\mathrm{d}s - \int_{-\infty}^{\infty} n(s).\sigma(s, \epsilon\eta(s)).U(s)J(s)\mathrm{d}s$$

At order 0, the classical Galin's equations are recovered. U, V are components of the displacement along the $x, \eta(x)$ curve, developing the equation at order 1, they satisfy

$$\begin{aligned} c_1U_1 &= -2\theta T_1 + \frac{1}{\pi}\int \frac{\eta(x)-\eta(s)}{x-s}T_2 ds - \frac{2}{\pi^2}\int_{-a}^{a}\int \frac{\eta(s)}{(x-s)(s-t)}dsT_1(t)dt \\ c_1V_1 &= \frac{1}{\pi}\int \frac{\eta(x)-\eta(s)}{x-s}T_1(s)ds \end{aligned}$$

that is the answer of the Galin's equations on $y = 0$ taking account of the convected derivation of the boundary conditions. These equations are quite different from the proposed in [4] due to no specification of particular boundary condition. The convected derivative $D_\eta\sigma.e_y$, leads to $D_\phi(\sigma.e_y) = \dot{\sigma}.e_y + \frac{\mathrm{d}}{\mathrm{d}x}(\eta\sigma.e_x)$. After the presentation of this development, some particular examples are investigated.

Some applications

Now we propose to present some applications of the problem of wear contact.

Case 1 : Applying the same reasoning than in [2], the influence of the viscous layer is studied and the role of the critical state σ_c is investigated. The viscous effect is equivalent to friction.

Case 2: For the same wear law, without the layer, the evolution of the boundary of the half plane is investigated. The Hilbert problem is solved based for different boundary conditions. Due to the critical value σ_c the area of contact is no longer symmetric, and the evolution of the boundary is given in a closed form.

Case 3. For cyclic loading, the solution is obtained in a asymptotic manner as proposed in [4], the influence of the critical value is discussed as well as the distribution of pressure under the punch.

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Towards an implicit finite wear formulation for non-smooth contact geometries

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Summary: An implicit finite wear algorithm based on dual mortar methods for surface-to-surface contact settings is presented at finite deformations and an algorithmic extension for the treatment of non-smooth contact geometries is given. Herein, general challenging aspects for the computation of non-smooth contact scenarios will be discussed.

A finite wear contact formulation for smooth geometries

Wear phenomena are of great importance for industrial applications due to their destructive impact on machines and strongly loaded components. They represent a process of material removal associated with frictional contact, which results in damage at the contact zone due to the accumulation of wear. The main parameters affecting the wear process are the contact stresses, the slip amplitude, the contact geometry and material properties. Archard's law [1] is a general phenomenological equation, which combines these influences within a simple wear constitutive law despite the existence of various different types of wear effects like abrasive, adhesive, corrosive and fretting wear. Therefore, it is the basis for wear modeling within this contribution.

In the past years, the computational treatment of wear phenomena was predominantly carried out by assuming only a small amount of wear caused by oscillatory relative displacements between two components [2]. Extensions to finite wear effects have first been realized for traditional node-to-segment contact formulations [3]. Nevertheless, mortar finite element methods have become the commonly accepted state-of-the-art approach for smooth geometries in computational contact mechanics [4, 5] and are utilized as underlying framework for this contribution.

Archard's wear law is reformulated to express the worn volume as interface displacements in negative normal direction and is weakly imposed following the segment-to-segment idea. The non-penetration and frictional sliding constraints are formulated as nonlinear complementarity problem, now also including consistent linearizations with respect to the additional nodal wear.

Constraint enforcement is realized via the Lagrange multiplier method employing so-called dual shape functions [6] and nonlinear complementarity functions, which decouple the interface constraints for each node and allow for straightforward condensation procedures for the discrete Lagrange multipliers. Dual shape functions are also used as testing functions for the weak enforcement of Archard's wear law, which opens up the possibility of applying an additional condensation procedure for the unknown nodal wear variables. This avoids any increase in the global system size of the linearized system of equations to be solved in each Newton-Raphson iteration step, while the full contact Lagrange multiplier information as well as the wear effects are included in the condensed system.

In addition, the underlying shape evolution problem is solved with the help of an Arbitrary Lagrangian-Eulerian (ALE) formulation, which guarantees proper mesh quality even if a significant amount of material is worn off. The shape evolution problem within the ALE step and the wear calculation within the classical structural step including contact are solved iteratively in a partitioned solution scheme until convergence is achieved.

Extension to non-smooth contact geometries

Despite the excellent performance of mortar methods in a classical smooth surface-to-surface contact scenario, unreasonably large penetrations will occur for non-smooth contact geometries such as corner-to-surface, edge-to-surface or edge-to-edge settings. Thus, a hybrid formulation for two-dimensional problems based on a point-to-line (PTL) approach and a mortar method will be presented in this contribution. Here, the PTL approach will be applied on geometrical corners to prevent large penetrations and the mortar method will be applied on smooth parts of the contact zone.

One of the main challenges of such a formulation is the distinction between real geometrical corners and kinks due to the typically only C^0-continuous finite element approximation. Furthermore, a proper transition law to switch between the PTL approach and the mortar method will be discussed.

The hybrid formulation will be included in the introduced finite wear framework and first results for non-smooth contact scenarios including wear effects will be illustrated. In addition, an outlook on the corresponding three-dimensional algorithm will be given.

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Modelling of wear of materials in terms of variational methods

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Summary: The objective of this work is to describe in detail variational settings for wear processes of materials by including wear debris between contacting solids and a clearance gap evolution during the process. Variational methods together with the finite element method are commonly used in computational contact mechanics.

Introduction

There is a great need to improving the wear-resistance of machinery component parts operating in contact conditions. In this contribution our aim is to obtain the complete set of governing relations describing mechanics of wear processes. Several generalizations of classical variational principles are presented in the case of contact problems for wearing out solids.

Variational methods in contact mechanics and predictions of contact stresses

For numerical solutions of contact problems with the aid of the finite element method one needs variational forms of governing equations. In computational contact mechanics, in variational formulations of deformations of contacting solids and in heat conduction problems the following principles are used: stationary total potential energy, virtual work and virtual power, quasi-variational principle of Hamilton, variational inequalities and their generalizations [1]. The heat is generated by the friction process. Several techniques are applied to fulfil the kinematic contact constraints and to calculate contact stresses, e.g. Lagrange multiplier method, penalty method, perturbated Lagrangian method, augmented Lagrangian method, mathematical programming methods and others [1, 2, 3].

Key factors in mechanics of wear

Wear is identified with a gradual removal of material from rubbing external surfaces of solids. Irreversible changes in bodies contours and an increase of the clearance gap between contacting solids are the main results of wear. An amount of the removed material can be estimated with the aid of wear patterns (profiles). A definition of the clearance gap between two contacting bodies includes deformations of the solids and the gap evolution as a result of the wear process [4].

The mechanism of wear involves formation of loose wear particles detached from the rubbing surfaces. The wear particles are trapped and accumulated between the sliding surfaces for some period of time. They form almost continuous intermediate layer which separates the rubbing surfaces. The wear particles transmit loads and displacements at the contact interface. The presence of wear particles between sliding surfaces affects friction and wear processes significantly [5].

Various constitutive models describe quasi-solid, quasi-fluid and granular-like behaviour of wear debris. In the discrete formulation each particle can be considered as an isolated solid body (granular material) and motion equations are applied to each particle. In the continuum formulation the wear debris are treated as a single two-dimensional layer. In this contribution, the layer is considered as a continuum with own morphology, kinematics and constitutive models (micropolar thermoelastic layer). Two governing equations define displacements and micro-rotations. Small deformations and small micro-rotations are considered in the layer [5].

Modelling of deformations and heat conduction in wearing out solids

Differential forms

The objective of wear process mechanics is to determine patterns (profiles) of external boundaries of wearing out solids, and to include an effect of wear debris on deformations of the rubbing bodies. Differential forms of governing equations for the contacting solids and the intermediate layer of wear debris are constructed from general balance laws: mass, momentum, moment of momentum, energy and entropy. Additional terms in the governing equations of the layer define a mass of wear debris supplied to the layer during the course of the wear process.

Variational forms

An effective way to solve wear problems could be given by variational formulations and an approximation with the aid of finite element method. A variational description of deformations of the contacting bodies is presented with the aid of the principle of stationary total potential energy. The total potential energy is defined in a deformed configuration of two contacting solids and the thin intermediate layer of wear particles. Furthermore, variational settings of the heat conduction (in the contacting bodies) and the mass continuity problem (in the layer) are discussed.
To solve the contact problems the incremental formulation and iteration procedures must be taken. We search such fields of displacements, temperatures and the mass intensity which guarantee the stationarity of the variational functionals at the given step of incremental approach and the iteration process. The stationarity condition of the total potential energy for any finite element leads to equations of displacements in the bodies and in the layer, and to equations of micro-rotations in the layer. The variational formulations and constrained methods (e.g. the Lagrange multiplier method, the penalty method, etc.) provide a powerful tool to solve the contact and wear problems.

Conclusions

In the contribution classical differential and variational formulations used in contact mechanics are extended by including an increase of the gap due to the wear process, and by taking into account the intermediate layer of wear debris. In this way the wear effects are introduced into the variational formulations of deformations and heat conduction problems. Proposed formulations can be useful in numerical analysis of machine parts subject to the wear process. An advantage of this approach is that more accurate models of wear can be easily implemented.

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Modelling of frictional cohesive contacts

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Summary: A general model for a large variety of cohesive contacts between visco-elastic bodies with friction is presented. The numerical solution includes a semi-implicit time discretisation and a spatial discretisation by SGBEM, leading to an efficient numerical implementation.

The general model for contact

We consider a bounded Lipschitz domain $\Omega\subset\mathbb{R}^d$ splitted into several bodies by internal interfaces Γ_C. The model includes the force equilibrium on the contact interface Γ_C:

$$[\![\sigma]\!]\vec{n}=0 \quad \text{with} \;\; \sigma:=\mathbb{D}e(\dot{u})+\mathbb{C}e(u) \tag{1a}$$

$$\sigma_\mathrm{n}-\kappa_\mathrm{n}(\zeta)[\![u]\!]_\mathrm{n}-\sigma_\mathrm{c}=0, \text{ with } \;\sigma_\mathrm{c}:=-\kappa_\mathrm{c}[\![u]\!]_\mathrm{n}^-, \tag{1b}$$

$$|\sigma_\mathrm{f}|<-\mu(\zeta)\sigma_\mathrm{c} \;\;\Rightarrow\;\; [\![\dot{u}]\!]_\mathrm{s}=0, \qquad |\sigma_\mathrm{f}|=-\mu(\zeta)\sigma_\mathrm{c} \;\;\Rightarrow\;\; \exists\lambda\geq 0:\;\; \sigma_\mathrm{f}=\lambda[\![\dot{u}]\!]_\mathrm{s}, \tag{1c}$$

$$\text{with } \;\sigma_\mathrm{f}:=\sigma_\mathrm{s}-\kappa_\mathrm{s}(\zeta)\big([\![u]\!]_\mathrm{s}{-}\pi\big), \text{ and } \;\sigma_\mathrm{s}:=\sigma\vec{n}-\sigma_\mathrm{n}\vec{n} \quad \text{with} \quad \sigma_\mathrm{n}:=\vec{n}^\top\sigma\vec{n},$$

and furthermore the flow rules for delamination and plastic slip on Γ_C:

$$\partial_{\dot{\zeta}}a_1\big([\![u]\!],\dot{\zeta}\big)+\frac{1}{2}\kappa_\mathrm{s}'(\zeta)\big|[\![u]\!]_\mathrm{s}{-}\pi\big|^2+\frac{1}{2}\kappa_\mathrm{n}'(\zeta)\big(\zeta,[\![u]\!]_\mathrm{n}\big)^2+N_{[0,1]}(\zeta)\ni \mathrm{div}_\mathrm{S}(\kappa_2\nabla_\mathrm{S}\zeta), \tag{2a}$$

$$\dot{\pi}\in N_{|\cdot|\leq\sigma_\mathrm{y}(\zeta)}\big(\mathrm{div}_\mathrm{S}(\kappa_1\nabla_\mathrm{S}\pi)-\kappa_\mathrm{s}(\zeta)\big([\![u]\!]_\mathrm{s}{-}\pi\big), \tag{2b}$$

$$\text{with boundary conditions on } \partial\Gamma_\mathrm{C} \;\; \nabla_\mathrm{S}\pi{\cdot}\vec{n}_\mathrm{S}=0, \quad \nabla_\mathrm{S}\zeta{\cdot}\vec{n}_\mathrm{S}=0, \tag{2c}$$

where $[\![\cdot]\!]$ denotes the jump of the quantity across Γ_C and $\vec{n}_\mathrm{S}$ denotes the normal to $\partial\Gamma_\mathrm{C}$. In the bulk, we consider the standard visco-elasticity (no intertia) supplemented with standard boundary conditions:

$$-\mathrm{div}\sigma=g \quad \text{in } \;\Omega\backslash\Gamma_\mathrm{C}, \qquad u=u_\mathrm{D}(t) \quad \text{on } \;\Gamma_\mathrm{D}, \qquad \sigma\vec{n}=f(t) \quad \text{on } \;\Gamma_\mathrm{N}. \tag{3}$$

In addition, we consider an initial-value problem by prescribing the following initial conditions

$$u|_{t=0}=u_0, \qquad \pi|_{t=0}=\pi_0, \qquad \zeta|_{t=0}=\zeta_0. \tag{4}$$

An example

We intend to illustrate the above general model on a multipurpouse example

The rate independent damage [1], with $a_1(\dot{\zeta}){=}{-}G\dot{\zeta}$ for $\dot{\zeta}{\leq}0$ and infinite otherwise, is considered for the fracture energy $G{=}1\mathrm{kJ\,m^{-2}}$. The stiffness function $\kappa_\mathrm{s}(\zeta)$ provides known cohesive zone models as described in [2], e.g. the Ortiz-Pandolfi-like model (OP), or the bilinear model (BL) :

$$\kappa_\mathrm{s}^\mathrm{OP}(\zeta){=}\kappa_\mathrm{s0}\frac{\exp\big(\gamma_p^{-1}(\varepsilon)-\gamma_p^{-1}(\alpha\zeta{+}\varepsilon)\big)-1}{\exp\big(\gamma_p^{-1}(\varepsilon)-\gamma_p^{-1}(\alpha{+}\varepsilon)\big)-1}, \;\;\text{or}\;\; \kappa_\mathrm{s}^\mathrm{BL}(\zeta){=}\kappa_\mathrm{s0}\left(\frac{\beta\zeta}{1+\beta-\zeta}\right), \tag{5}$$

where γ_p^{-1} is the inverse of the upper incomplete gamma function $\gamma_p(x)$. In the test we use κ_{s0}=10GPa m^{-1}, p=3, α=0.99, ε=0.005m, and β=0.10598, having the same maximal cohesive stresses in both models.

The friction coefficient μ is considered damage dependent [3]: $\mu(\zeta)=\mu_0(1-\zeta)^q$, with μ_0=0.022, q=1 or 4.

With the plastic slip, the scenario includes the plastic slip π triggered upon reaching the yield stress σ_y=1.25MPa with possible damage evolution due to plastic hardening with κ_H=0.5GPa m^{-1}.

We took a simple geometry from Figure 1(a) loaded by displacement loading as shown in the graph. In the example, only the tangential component plays a role in damage,

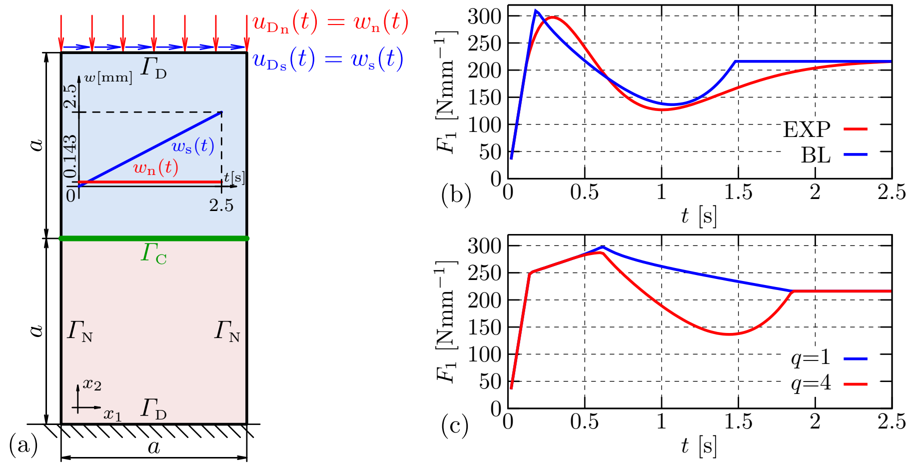

Figure 1: The example: (a) geometry and loading: a=200mm, E=200GPa, ν=0.3. Time dependence of the total tangential force F_1 applied at $x_2=2a$ considering friction and: (b) both BL and OP cohesive laws, with no plastic slip and q=4, (c) BL model only, with plastic slip.

because of compression with κ_c=3.846TPa m^{-1}.

The overall response of the model to the loading can be seen in Figure 1(b,c). We used both proposed cohesive models and both choices of p in the friction function. It should be noted that the greater value of p causes a postponed frictional response, because $\mu(\zeta)$ for ζ close to one is vanishing compared to q=1. If we use the bilinear cohesive model and also linear function μ (q=1), the structure response is also (almost) piecewise linear as seen in Figure 1(c). Additionally, the graphs in Figure 1(c) correspond to the use of the BL model with plastic slip.

Acknowledgement. VEGA 1/0078/16 (Slovakia), MAT2012-37387 (Spain)

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On a Griffith condition for the stick-slip boundary

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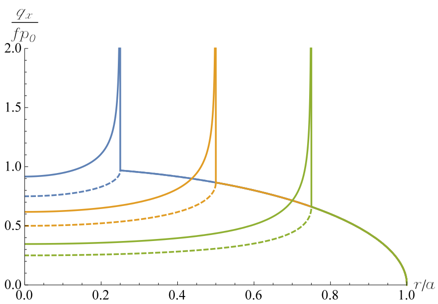
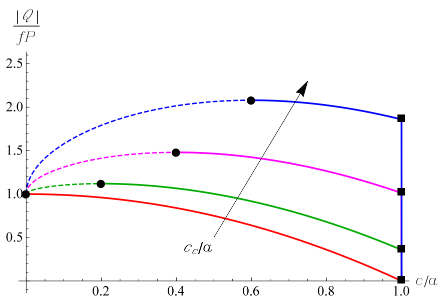
Summary: The standard Cattaneo-Mindlin problem is studied in the case the stick-slip boundary is found via an energetic criterion (fracture energy), which leads to similarities with the singular solution for adhesive contact problems. A double Cattaneo-Mindlin model is adopted to show that the fracture energy is related to a certain integral of the slip-weakening friction law.

Abstract

Classically frictional contact is modelled using Amonton-Coulomb law which implies proportionality between normal and tangential load. In some cases two friction coefficients are adopted to model the frictional strength that has to be overcome in a contact to initiate sliding, after which, during relative motion, a lower friction coefficient is adopted. Recently a large number of experimental results have been published regarding the inception of slip and dynamic ruptures of frictional interfaces [1–5]. The experimental set-up consists in two blocks of PMMA that are pressed together and tangentially loaded. As PMMA is a transparent material, laser sheet is adopted together with a fast camera to visualize real time changing of contact area and thus slip. In particular in [5] is shown that fracture mechanics well describes the shear tractions distribution near the tip of the traversing slip front. Classically Cattaneo [6] and Mindlin [7] solved the case in which two similar elastic bodies, which gap can be described by quadratic functions, are first pressed together and then tangentially loaded (quasi-statically) using one coefficient of friction $f = f_d$.



(a)



(b)

Figure 1: (a) Shear traction distributions $\frac{q_x}{fp_0}$ for the classical Cattaneo-Mindlin solution (dashed lines) and superposed the singular solution (solid lines) for the ratio $c/a = [0.25, 0.5, 0.75]$, (b) Q/fP for Hertzian contact plotted against c/a for different values of the critical dimension of the stick area $c_c/a = [0, 0.2, 0.4, 0.6]$. For each curve the minimum Q/fP at which partial slip commences is indicated with a black square, while the maximum value of the stable branch is indicated with a black circle and coincides with the abscissa $c/a = c_c/a$.

We revisit the standard Cattaneo-Mindlin problem and we derived the case in which the stick-slip boundary is defined using an energetic criterion [8], in which the key parameter is the fracture energy G, while in the slip zone the Amonton-Coulomb law still holds. The tangential problem is shown to be similar to the normal problem in which adhesion is considered, as in the JKR solution (see Fig. 1a). We show that the stick area shrinks up to a certain minimum (grater than zero) at which full sliding occurs leading to an apparent friction coefficient f_{app} which is grater then f_d (see Fig. 1b). We show that for a given fracture energy G, f_{app} depends on the geometrical details of the contact pair [9]. We plot the curves tangential load vs stick area for Hertzian, power-law and sinusoidal profile [9]. Further we solve the Cattaneo-Mindlin problem for an Hartzian profile in which a slip-weakening friction law is assumed [10]. Under the assumption that the shear traction distribution can be approximated via a superposition of 2 classical Cattaneo-Mindlin solutions we prove that if the transition from static to dynamic friction is sharp enough a limiting solution similar to the JKR solution for adhesive problem is retrieved (but in the tangential direction) and the energy involved is shown to be equivalent to the shear fracture energy defined by Abercrombie and Rice in [11]

$$W = \int_0^{+\infty} (f(u) - f_d)\, p du, \tag{1}$$

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A bipotential method coupling contact, friction and adhesion

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Summary: Following the general works of Gery de Saxcé in the domain, a bipotential formulation is presented here for a model of interface coupling unilateral contact, friction and adhesion (the RCCM model). To deal with the non-associated character of the Coulomb friction law with adhesion, a specific potential is given and treated. A numerical method is proposed and tested on several examples of delamination.

Introduction

In order to simulate the behaviour of complex interfaces, a cohesive model (RCCM) coupling adhesion, friction and unilateral contact was proposed by Raous et al in [1]. This model gives a smooth transition from total adhesion to the usual Coulomb friction law with unilateral contact. Because of the implicit character of the Coulomb friction law, the notion of superpotential with normality rule, cannot be used anymore, the law is non-associated. To overcome this non-associated character, when using the bipotential concept proposed by De Saxcé and Feng [2] and used by Berga-De Saxcé [3] in plasticity, a specific bipotential is proposed and the complete formulation is given. A numerical method is implemented in the finite element code SYMEF at Bechar (Berga-Terfaya [4]). Efficiency is showed on two examples.

Bipotential coupling contact, friction and adhesion

This formulation is based on the works of Raous et al [1] [5] on the soft coupling between adhesion and frictional contact. A change of variable for the sliding velocity is introduced and the following bipotential (1) is constructed. It is composed of two parts, one controlling the interface law and the other one controlling the adhesion evolution:

$$b(-\dot{\boldsymbol{u}},\dot{\beta},\bar{\boldsymbol{R}},\boldsymbol{G}_{\beta}) = I_{K_{\bar{\mu}}}(\bar{\boldsymbol{R}}) + I_{\Re^-}(-\dot{u}_n) + \mu(1-\beta).\bar{R}_n\|-\dot{\boldsymbol{u}}_t\| + \frac{\alpha\dot{\beta}^2}{2} + I_{C^-}(\dot{\beta}) + \frac{(\boldsymbol{G}_{\beta}^-)^2}{2\alpha} \tag{1}$$

where $\dot{\boldsymbol{u}}$ is the relative velocity, β the intensity of adhesion taking value between 0 and 1, $\bar{\boldsymbol{R}}$ the contact force and $\boldsymbol{G}_{\beta}$ the thermodynamic force associated to the state variable β. The parameter α is the adhesion viscosity and μ is the friction coefficient. $K_{\bar{\mu}}$ is the Coulomb's cone, I_s denotes the indicator function of the specified sets S. In (1), the indicator function $I_{\Re^-}$ imposes the unilateral conditions and I_{C^-} imposes the condition $\dot{\beta} \leq 0$ which means that the evolution of the intensity of adhesion is an irreversible and dissipative process and that the adhesion can only decrease. It has been shown that this bipotential verifies the suitable properties of bi-convexity and satisfies the Fenchel inequality. Then the contact laws with adhesion and the equation for the evolution of β, which are explicitly given in [1] and [5], are deduced through the state and complementarity laws expressed in term of implicit subnormality rules or differential inclusions:

$$-\dot{\boldsymbol{u}} \in \partial_{\bar{\boldsymbol{R}}} b(-\dot{\boldsymbol{u}},\bar{\boldsymbol{R}}); \quad \bar{\boldsymbol{R}} \in \partial_{-\dot{\boldsymbol{u}}} b(-\dot{\boldsymbol{u}},\bar{\boldsymbol{R}}) \tag{2}$$

where $\partial_x b(-\dot{\boldsymbol{u}},\bar{\boldsymbol{R}})$ denotes the sub-differential of b with respect to the variable x. A method based on augmented Lagrangian formulation adapted to variational inequalities is used. A saddle point problem is obtained and it is solved by using an Uzawa algorithm. A prediction-correction process combined with projection leads to a sequence of minimization problems under constraints which are reduced to regular minimization problems when a Lagrange multiplier is introduced.The frictional contact problem with adhesion is then treated in a reduced system [2,4].

Numerical results

Validation and efficiency of the method is showed on two benchmarks simulating delamination. The presented formulation is compared with the previous one on the RCCM model developed by Raous et al and implemented in the GYPTIS90 code (LMA Marseille) [5]. The adhesion intensity β, the tangential displacement $\boldsymbol{u_t}$ and the normal displacement u_n are presented along the interface.

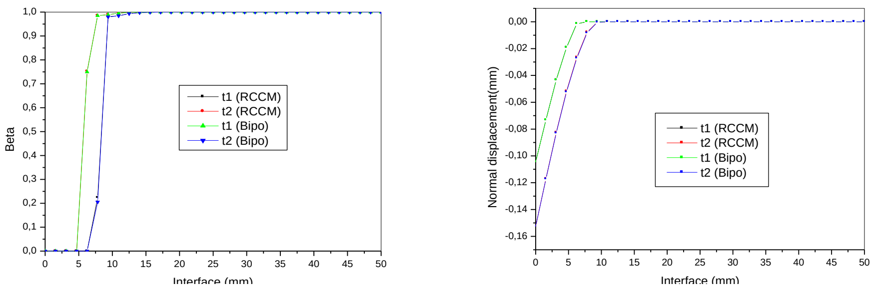

Figure.1 : *Delamination of a thin layer of aluminium submitted to vertical loading: evolution of the adhesion intensity β and of the normal displacement* u_n *along the interface*

The two approaches are compared on the numerical results presented on Figure 1. A good agreement can be noted.

Conclusion

In this work, a bipotential formulation for the RCCM model, coupling adhesion and friction has been theoretically investigated and numerically implemented. It has shown that on the interface, the frictional contact law with adhesion described by a non-associated sliding rule and its inverse are obtained by applying the normality rule to a single scalar-valued function called a bipotential, which leads to a single displacement variational principle and a single inequality. The ability of this framework was illustrated on the numerical simulations where the model is tested and compared with the previous formulation [1][5] on two benchmark simulating delamination.

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The two fundamental fracture modes of one-dimensional bars

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Summary: A variational approach based on incremental energy minimization shows that materials can break according to two fundamental fracture modes, ductile and brittle. Typical representatives of the two modes are steel and non-reinforced concrete. The overall behavior of a bar subject to monotonic loading up to rupture is governed by two non-dimensional numbers, each of which is a combination of material parameters.

In recent years, a theoretical research developed by the author and co-workers, [1]–[4], produced an algorithm reproducing the evolution of a body from the initial unstressed state up to final rupture. At the present stage of the research, only results for straight bars with constant section are available. The method is flexible enough to reproduce, with surprising accuracy, the response of materials of a very different nature. The starting point is to assume a strain energy of the form

$$E(\epsilon,\gamma) = \int_0^l \big(w(\epsilon(x)) + \theta(\gamma(x)) + \tfrac{1}{2}\alpha\gamma'^2(x)\big)\,dx \tag{1}$$

where ϵ and γ are the elastic and inelastic parts of the deformation, and w and θ are the corresponding energy densities. The last term in the integrand function is a non-local term depending on the derivative of γ, whose purpose is to stabilize the softening branch of the response curve. If no external forces are applied, the total energy of the bar is $E(\epsilon,\gamma)$, and the role of the load is played by the elongation β of the bar, which is controlled by prescribing the axial displacements at the endpoints.

The response to a given load process $t \mapsto \beta_t$ is determined by incremental energy minimization. At a given time t the current values of ϵ and γ are supposed to be known, and the problem is to determine the strain rates $\dot\epsilon$ and $\dot\gamma$ which minimize the variation of the energy under a prescribed variation $\dot\beta$ of the load. An approximated solution is obtained by solving a sequence of incremental problems, in which each solution, extrapolated to a finite time interval, is used as the initial condition for the next problem. With this procedure, the entire response curve of bars made of any specific material can be reproduced by ad hoc analytical forms of the functions w and θ.

The present communication anticipates the part of the book chapter [5] which deals with the two different fracture modes observed in the tensile tests. In the response curve of a bar subject to monotonic loading, the initial elastic regime is followed either by sudden fracture or by a more or less protracted inelastic regime. In the latter, the axial force either increases at a lower rate than in the elastic response (*hardening regime*), or decreases (*softening regime*). In general, the response curve consists of an alternation of hardening and softening regimes. In the softening regime, due to the presence of the non-local energetic term, the inelastic deformation can be either *full-size*, that is, diffused all over the bar, or *localized*, that is, concentrated on a more restricted zone.

At the end of the response curve, fracture may take place in two very different modes. The *brittle fracture mode* is characterized by progressive strain localization, with a consequent increase of the negative slope of the response curve. Rupture takes place when the localization of the plastic strain becomes extreme and the slope of the curve becomes infinite. On the contrary, in the *ductile fracture mode* the distribution of the inelastic strain tends to become uniform over the bar, and the intensity of the axial force tends to zero. The most evident difference between the two fracture modes is that the slope of the stress-strain curve at rupture is infinite for brittle fracture and zero for ductile fracture. The first mode is typical of steel and other metallic materials, and the second is typical of non-reinforced concrete and other amorphous materials.

With a few exceptions [6, 7], these differences in the behavior at rupture did not receive much attention in the literature. After some partial results exposed in [2, 3], a more precise conclusion has been illustrated in [5]. It is shown there that the bar's response is determined by two non-dimensional parameters

$$l^2\theta''_{min}/\alpha\,, \qquad \theta''_{min}/K\,, \tag{2}$$

in which $K = w''(\varepsilon)$ is the Young modulus of the material and θ''_{min} is the minimum of the inelastic energy density $\theta(\gamma)$ for γ in $(0, +\infty)$. The first parameter controls the transition from full-size to localized solution, and the second controls the occurring of brittle fracture. The two parameters are independent, and with suitable choices of the functions θ and w it is possible to reproduce the response curves of bars which break just at the onset of the inelastic deformation, of those which break in a hardening regime or after a more or less extended softening regime, as well as of bars which do not break at all, but collapse by ductile fracture.

At first sight, this identification procedure for θ and w may look like a sort of curve fitting. In reality, there are perspectives for rendering the theory predictive. Indeed, if the correlations between the shapes of the macroscopic energy densities w, θ and the microscopic structural properties of matter were known, it would be possible to design materials with the desired macroscopic properties.

In the proposed communication, the results of some numerical simulations will be discussed and compared with the experimental data from tensile tests on steel bars and non-reinforced concrete bars.

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Cracks closure in thin plates and shells: mixed problems and analytical solutions

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Summary: The problem of through cracks and narrow slits closure in thin-walled structural elements under bending loading is being studied in a two-dimensional formulation by means of the classical theories of plates and shells as well as contact model along the line. The report provides an overview of analytical solutions of some mixed contact problems for cracks with interacting edges.

Model of crack closure and mixed problem

An incomplete in the thickness plate (shell) crack edge contact during bending under the Kirchhoff hypothesis and within the framework of geometrically linear approach is interpreted as a closure of the edges of the cut in the front-face of the plate. Upon such consideration the unknown two-dimensional contact area is approximated by lines the configuration of which in the front of the plates is generally unknown. Boundary conditions of nonpenetration of crack edges and the relationship of forces and moments on the cut corresponding to this model have been recorded [1, 2].

A class of structurally nonlinear boundary value problems of the theory of plates and shells with coupled boundary conditions at the cuts that describe an interaction of crack edges under the conditions caused by bending has been formulated. In case of plate bending the boundary problem is following:

$$\Delta\Delta\varphi = 0, \quad \Delta\Delta w = 0, \quad (x, y) \in \mathbf{R}^2 \setminus L;$$

$$N_y = 0, \quad M_y = -m(x), \quad [u_y] > h\,|[\theta_y]|, \quad x \in L_1,$$

$$[u_y] = h\,|[\theta_y]| > 0, \quad M_y = hN_y \operatorname{sgn}[\theta_y] - m(x), \quad N_y \le 0, \quad x \in L_2,$$

$$[u_y] = 0, \quad [\theta_y] = 0, \quad hN_y \pm M_y \le 0, \quad x \in L_3,$$

$$N_{xy} = 0, \quad Q_y^* = 0, \quad x \in L = L_1 \cup L_2 \cup L_3;$$

$$N_x = N_{xy} = N_y = 0, \quad M_x = M_{xy} = M_y = 0, \quad (x, y) \to \infty.$$

Here φ is the stress function and w is the deflection of the plate, Δ is two-dimensional Laplace operator, $2h$ is plate thickness, $[u_y]$ and $[\theta_y]$ are the crack opening in the shell middle surface and the jump in the angle of rotation of the normal, N_{ij}, M_{ij}, and Q_y^* are the membrane forces, moments, and generalized shear force, m – given bending load, L is cut on the x-axis.

The theoretical questions of existence, uniqueness and smoothness of solutions of such problems in Sobolev spaces have been investigated by means of the theory of variational inequalities [3].

Using fundamental solutions and Green matrix, boundary problems have been reduced to the systems of singular (by Cauchy) integral equations regarding the derivatives of jumps of vector component of displacements and angles of normal rotation on the cut lines with additional conditions to find the size of the contact outlines.

Analytical results

This report separately highlights mixed problems for the rectilinear cut, solutions of which are expressed analytically through quadratures or elementary functions and if the regular kernels are available – in parametric form through asymptotic series. Among them there are: the problem of closing the crack under bending constant-sign moment on the cut [1, 2], which generalizes the results [4, 5], mixed problem of partial closure of cracks in various facial surfaces under alternating moment, the problem with fixed points of demarcation of the boundary conditions for crack connected with a slit [6, 7], nonlinear mixed problems about interaction of edges of narrow slit in the bent plate, the problems of crack edges contact in plates and shells under combined tension and bending [8, 9]. Based on analytic and asymptotic solutions of these problems, the crack tip stress intensity factors, the configuration of the contact areas and contact reaction on the closed cuts have been obtained. With the use of energy criterion of the linear fracture mechanics under the combined tension and bending the influence of crack-like defects closure on limiting equilibrium of plates and shells has been investigated.

Conclusion

Despite the limited capacity of the classical plates and shells bending theory, the proposed approach allowed to avoid kinematic contradictions associated with mutual penetration of opposite surfaces of cracks in compression zones. Analytical assessments of limiting equilibrium of defective plates are suitable for any value of tensile and bending loads.

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Elastodynamic frictional sliding of an elastic layer on a rigid flat: stick-slip pulses and opening waves

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An elastodynamic frictional problem for an elastic layer sliding on a rigid flat was solved numerically for a wide range of friction coefficients and Poisson's ratios. Slip localization was observed in the stable regime, fast opening waves emerged in the unstable regime leading to the inversion of the frictional force.

Introduction and methods

Elastodynamic frictional problems, even for simple Coulomb friction, present a great challenge both from mathematical and computational points of view. These problems arise when two different materials (bi-material interface) slide one on another. Renardy [1] was the first who demonstrated that for certain values of Coulomb's friction coefficient, the sliding between an elastic half-space on a rigid flat becomes ill-posed, similar results were obtained later in [2]. The same year, Adams [3] obtained a more general result for two deformable elastically different materials, for certain material and interface parameters the frictional sliding problem is ill-posed in the sense that any perturbation on the interface grows exponentially with the growth rate proportional to the perturbation wave-number, thus this problem is unresolvable in the context of the classical continuum which does not posses a characteristic length. This result was analyzed theoretically and numerically in [4], and it was shown that an experimentally motivated regularized friction law [5] is able to solve most of the problems related to the ill-posedness. Later, Adams [6] theorized that for the case of rigid-deformable contact even in the stable regime the slip may localize in supersonic stick-slip pulses.

We study the transition from stick to slip and quasi-steady slip on the frictional interface between an elastic layer and a rigid flat. The interface is governed by Coulomb friction and regularized friction (Prakash-Clifton regularization [5]). Analytical results are complemented with implicit and explicit finite element simulations.

Main results

In the considered system, two sliding regimes can be distinguished: (1) an ill-posed regime (flatter instability) for the friction coefficient higher than the critical value $f = 1$ and smaller than the rapidly increasing function f_c depending on the Poisson's ratio: $1 \le f \le f_c(\nu)$, and (2) a well-posed regime for $f < 1$ and $f > f_c(\nu)$ [2]. We demonstrate that the flutter instability in the regime (1) is attenuated due to the manifestation of eigen modes of the finite thickness elastic layer working as a waveguide for elastic waves. It results in localization of the stick-slip pulses which transform to stick-slip-opening pulses, rather similar to Schallamach waves, but much faster ones (Fig. 1,a). This prominent behavior of the transition from the stick to slip results in a fast transformation of the entire stored elastic energy into kinematic energy of elastic waves. This process leads to a rapid drop of the frictional resistance of the system and even may lead to the inversion of the frictional force [7]; it happens when the contact interface is brought further than the top surface at which a constant tangential velocity is prescribed. At longer time scales, a classical stick slip behavior is observed (which is rather surprising

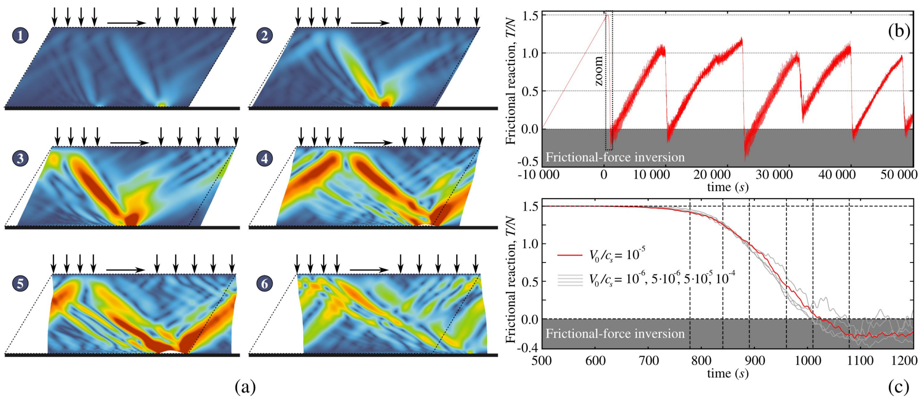

Figure 1: (a) Norm of the material points velocity during the transition from stick to slip and the formation of the opening wave; globally measured ratio of the tangential to normal force: (b) on a long period, (c) during the transition from stick to slip.

as a simple Coulomb friction was used in the interface). The apparent static friction coefficient does not exceed the unity $f^{\text{app}} \approx 1$, which is smaller than the local friction coefficient (Fig. 1,b), the kinematic friction coefficient is even smaller. The velocity of slip pulses was found to be intersonic. However, for high Poisson's ratios $\nu > 0.45$ we observe also supersonic(!) stick-slip-opening pulses, whose existence was predicted in [6]. In the "stable" regime $f < 1$ a rather similar behavior was observed for the entire range of Poisson's ratios: localization of supersonic stick-slip pulses and the variation of the apparent frictional resistance with the sliding velocity, both results in accordance with [6]. These numerical results are also in good agreement with analytical results, which were obtained in a fashion similar to earlier studies of [4, 8] and which generalize earlier results [2, 6].

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Contact calculation in elastic multibody gear systems using different kind of model order reduction methods

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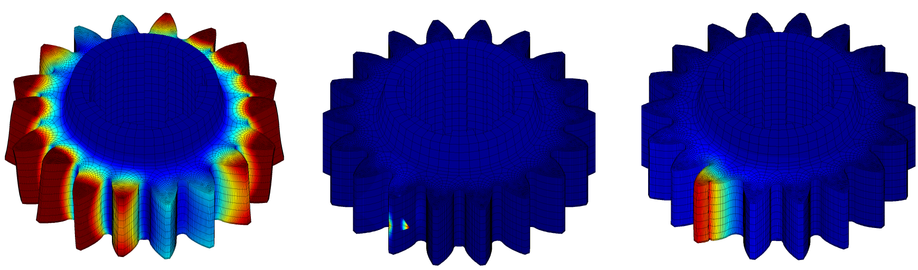
Summary: Reduced elastic multibody gear systems are modelled using different model order reduction techniques. These strongly affect contact calculation in highly dynamical gear contact situations. The differences in contact forces and thus, the kinematics of the results are shown and compared to a corresponding FE-model.

Gear systems modelled as reduced elastic multibody system allow a faster time integration of the dynamic problem compared to a comparable FE-model [1]. Gears are normally reduced from a couple of hundred thousands degrees of freedom (DOF) to a couple of hundred DOFs using model order reduction techniques. Classically, modal truncation or static modes enriched subspaces such as the Craig-Bampton method [2] are applied, see Figure 1. Both have advantages and disadvantages and strongly affect contact force calculation. For the gear contact problem, specifically designed modes such as snap-shots from an FE-simulation or constrained modes can be used to benefit the quality of the result as well as the computational effort. A modally reduced elastic gear can be integrated much faster in time as a comparable FE-model, but does not allow local approximations in the contact area [3]. A reduction technique with constrained modes does allow the geometrical decomposition of the contact area, but is computationally expensive. Such a reduced gear is well suited to obtain local deformations needed for reasonable results if a hard contact calculation is used.

Time integration is performed in modal space, but contact must be calculated in nodal space. Therefore, the nodes of the teeth in contact are transformed from modal in nodal space in every time step. Only the nodes of the teeth in contact are used for transformation in order to minimize numerical effort. A master-slave approach with a 4-node master segment is used to determine the penetration [4]. When a penalty contact is calculated, the contact force can be calculated in a reliable manner, but the



(a) (b) (c)

Figure 1: Different kind of mode shapes: (a) eigenmode, (b) constrained mode and (c) static mode from FE.

penalty parameter strongly depends on number and kind of used mode shapes. If a hard contact is applied, the contact area must allow small local deflections in order to obtain a reasonable contact force [5].

A gear pair will be used to show the dependency of the contact force on the used mode shapes in highly dynamical gear contact simulations. Here, a penalty approach will be used. Next, a hard contact will be applied together with local deformation mode shapes and the results will be compared against each other. For small integration times, a finite element reference result of a comparable FE-model will be taken into account to compare contact force, kinematics and numerical effort.

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Indentation response of magneto-electro-elastic materials under frictional contact

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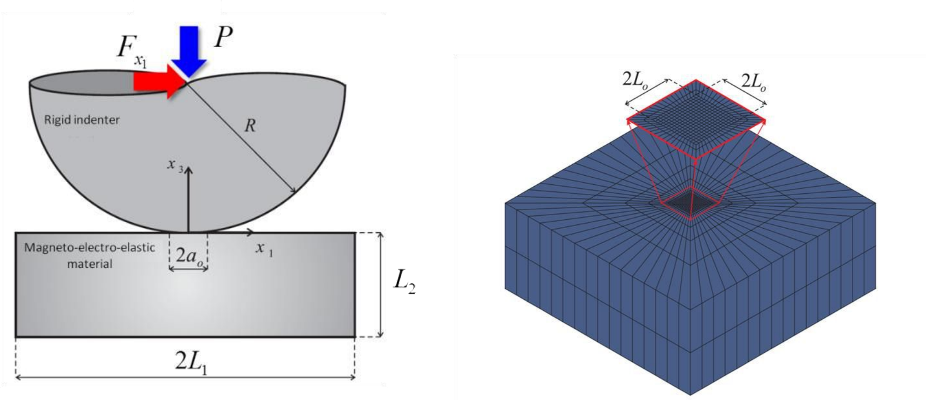
Summary: This work presents a numerical framework which deals with the general contact problem for coupled magneto-electro-elastic materials and allows to study the indentation response of multifield materials. Numerical results reveal that the conductivity parameters, friction or tangential loads conditions have a significant effect on the elastic, electric and magnetic fields.

Introduction

Coupled magneto-electro-elastic (MEE) materials are highly demanded for relevant technological applications. However, this topic of research has been treated only in some analytical works [1-6]. Analytical solutions lack the generality of numerical methodologies being restricted typically to simple geometries, loading conditions, idealized contact conditions and mostly taking into account transversely isotropic material symmetry with the symmetry axis normal to the contact surface. Based in [7], this work presents a numerical procedure for the three-dimensional frictional contact modeling of anisotropic multifield magneto-electro-elastic materials. An orthotropic frictional law is considered, so anisotropy is present both in the bulk and on the surface. The methodology uses the boundary element method with explicit evaluation of the fundamental solutions in order to compute the magneto-electro-elastic influence coefficients. The contact model is based on an augmented Lagrangian formulation and uses an iterative Uzawa scheme of resolution. Conducting, semi-conducting and insulated electric and/or magnetic indentation conditions, as well as orthotropic frictional contact conditions are considered.

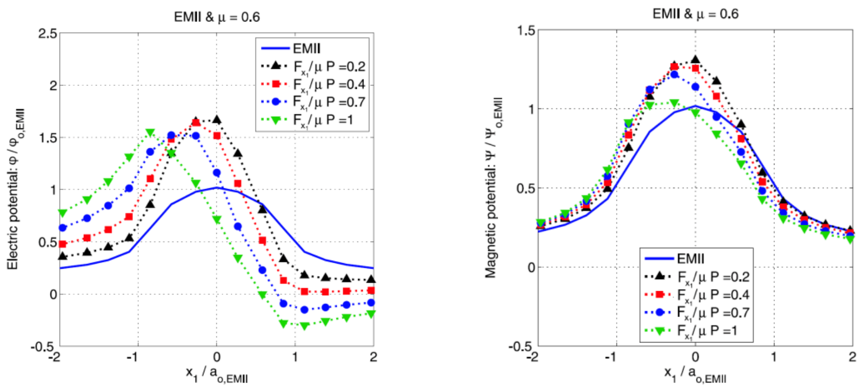
Results and conclusions

The methodology is validated by comparison with benchmark analytical solutions [2], and additional exploration examples are presented and discussed in detail. Results reveal that magneto-electric material coupling, conductivity, frictional and tangential load conditions lead to a significant effect on the indentation response of these materials. One of these examples is showed in Fig. 1 and Fig. 2. In Fig. 1 (a), the physical setting is presented and Fig.1 (b) shows the boundary element mesh details. The influence of tangential load conditions on the electric potential and the magnetic potential are presented in Fig. 2 (a) and Fig. 2 (b), respectively, for an electric and magnetic insulated indentation (EMII). They reveal that the mentioned aspects have an important influence not only in the values of the electric and magnetic potential, but also on their distributions and the maximum of electric and magnetic potential values.



(a) (b)

Figure 1: (a) Rigid indenter over a magneto-electro-elastic domain. (b) Boundary element mesh details.



(a) (b)

Figure 2: Normalized indentation and tangential load response distributions as function of the friction coefficient for EMII: (a) electric potential, (b) magnetic potential.

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Two-domain contact model of volumetric actuators

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Summary: We propose a framework for modeling *volumetric actuators*. Finite element implementation details and numerical examples are presented.

Volumetric actuators

Volumetric actuators are a specific class of actuators whose output forces are generated by micro-actuators distributed over their volumes and working in parallel. Well-designed volumetric actuators have maximum output forces proportional to the number of micro-actuators in their interior, and so to their volumes. This is unlike the usual actuator designs, e.g., hydraulic and pneumatic cylinders or piezoelectric crystals, whose forces are proportional to their cross-sectional areas, not volumes.

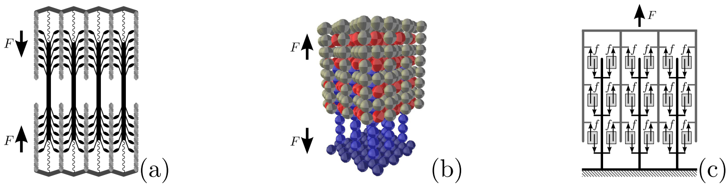

Figure 1: Volumetric actuators: (a) sarcomere, (b) modular-robot, (c) schematic.

A well known example of such systems is the sarcomere [1], which is a fundamental active subunit of animal muscles. In Fig. 1a, a schematic of the sacromere is presented, with heads of myosin molecules (black) working as micro-actuators, pulling adjacent actin filaments (gray), and producing an overall contracting force. Quite similarly, an artificially designed modular-robotic structure can possibly work as a two-directional actuator [2], see Fig. 1b.

Finally, Fig. 1c shows the general principle of operation of a (linear-motion) volumetric actuator, with two structures pushed/pulled in opposite directions with an overall force F—a sum of micro-actuations of magnitude f each. In order to enable finite extensions, each micro-actuator is realized as a contact system that enforces a slip between two constituent parts.

Two-domain contact model

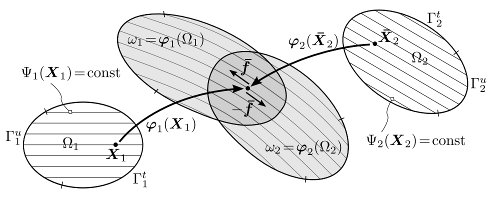

Figure 2: Continuum mechanics model.

In the presented macroscopic actuator model, two inter-penetrating bodies, Ω_1 and Ω_2, move against each other along some permitted directions (isolines $\Psi_i = \text{const}$), see Fig. 2. We assume a possible coupling between the allowed finite deformations and micro-actuation characteristics. Neglecting body forces, the weak form for two interacting actuator parts reads

$$G(\boldsymbol{\varphi}, \delta\boldsymbol{\varphi}) = G_1(\boldsymbol{\varphi}, \delta\boldsymbol{\varphi}_1) + G_2(\boldsymbol{\varphi}, \delta\boldsymbol{\varphi}_2) + G_c(\boldsymbol{\varphi}, \delta\boldsymbol{\varphi}) = 0, \tag{1}$$

where the variations $\delta\boldsymbol{\varphi} = \{\delta\boldsymbol{\varphi}_1, \delta\boldsymbol{\varphi}_2\}$ vanish on Γ_i^u, the weak forms

$$G_i(\boldsymbol{\varphi}, \delta\boldsymbol{\varphi}_i) = \int_{\Omega_i} \frac{\mathrm{D}W_i}{\mathrm{D}\boldsymbol{F}_i} \cdot \nabla\delta\boldsymbol{\varphi}_i dV_i - \int_{\Gamma_i^t} \boldsymbol{T}_i^* \cdot \delta\boldsymbol{\varphi}_i dS_i, \ \ i \in \{1, 2\} \tag{2}$$

are derived from (hyper)elastic potentials W_i with deformation gradients $\boldsymbol{F}_i = \nabla\boldsymbol{\varphi}_i$,

$$G_c(\boldsymbol{\varphi}, \delta\boldsymbol{\varphi}) = \int_{\bar{\Omega}_1} \left[\rho\left(\Psi_1(\boldsymbol{X}_1) - \Psi_2(\bar{\boldsymbol{X}}_2)\right) \frac{\partial\Psi_2}{\partial\bar{\boldsymbol{X}}_2} (\nabla\bar{\boldsymbol{\varphi}}_2)^{-1} + \bar{f} \frac{\boldsymbol{F}_1\nabla\bar{\Psi}_1}{\|\boldsymbol{F}_1\nabla\bar{\Psi}_1\|}\right] \cdot (\delta\boldsymbol{\varphi}_1 - \delta\boldsymbol{\varphi}_2)\, dV_1 \tag{3}$$

is the *contact part* of the weak form (1), in which $\bar{\boldsymbol{X}}_2$ is a point of Ω_2 such that $\bar{\boldsymbol{\varphi}}_2 = \boldsymbol{\varphi}_2(\bar{\boldsymbol{X}}_2)$ coincides with the point $\boldsymbol{\varphi}_1(\boldsymbol{X}_1)$ (note here that volumetric contact search is needed), $\bar{\Omega}_1 = \boldsymbol{\varphi}_1^{-1}(\omega_1 \cap \omega_2)$, and $\bar{\Psi}_1$ is so defined that $\nabla\bar{\Psi}_1$ are parallel to the isolines of Ψ_1. The first term in (3) is the enforcement of the condition that corresponding isolines of Ψ_1 and Ψ_2 must coincide. The second term imposes actuation slip of intensity $\bar{f}$ that is tangent to a deformed isoline $\nabla\bar{\Psi}_1$.

The above formulation is expressed in the same spirit as it is done for conventional contact formulations, see e.g. [3], which is particularly suitable for the finite element method implementation. The necessary FE procedures have been implemented and derived using the symbolic system *AceGen*, and the subsequent FE analyses were performed in the *AceFEM* environment [4].

Exemplary 2D FEM results

The results of an exemplary analysis of a two-dimensional contracting actuator are shown in Fig. 3. The actuator consists of two 20×10 mm^2 rectangles, clamped at opposite sides, with the initial 5 mm overlap. A plain-strain hyperelastic model for both parts is assumed, with the Young's modulus $E = 1$ MPa and Poisson's ratio $\nu = 0.4$.

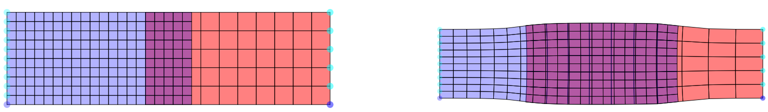

Figure 3: Left: initial mesh ($\bar{f} = 0$). Right: deformed mesh ($\bar{f} = 0.03\,[\mathrm{N/mm^3}]$).

In Fig. 4, the non-uniform deformation with horizontally-aligned stress gradients in the contact zone is presented, which is characteristic of volumetric actuators.

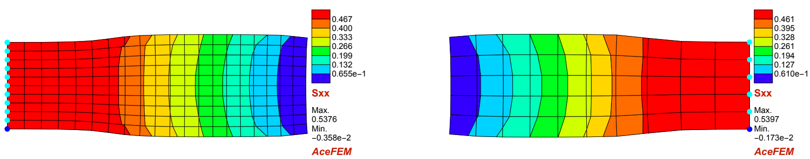

Figure 4: S_{xx} stress field in the deformed configuration for the left and right part of the actuator ($\bar{f} = 0.03\,[\mathrm{N/mm^3}]$).

Acknowledgement Work supported by the project "Micromechanics of Programmable Matter" (contract no. 2011/03/D/ST8/04089 with the National Science Centre in Poland

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Multi-numerics and multi-physics modeling of multiscale contact problems

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This work presents a multi-physics and multi-numerics modeling of contact problems. Our approach aims to improve contact modeling, by taking into account different contact models by means of suitable partitions of contact relations. It also uses the multimodel and multiscale Bridging Domain H/Arlequin Method to ease the numerics of the contact problem. The relevance of this seemingly new approach is exemplified by numerical tests.

Introduction

Contact problems are not only omnipresent, nonlinear and irregular, but also multi-physics, localized and multi-scale. In the literature, one can find many works centered on the nowadays quite mature modeling and simulation of macroscopic contact problems, handling the first three contact characteristic points, listed above (see e.g. [1] and [2]). In the contrary, relatively few works have been devoted to the three others issues.

One however can observe that during the last ten years and given the willingness of engineers to go a step forward on mastering not only delicate contact phenomena like wear, but also thermal and electrical resistance (e.g. for coating, design of MEMS and NEMS and bio-inspired design), there is a significant increase of the computational contact mechanics community interest for multi-physics and multi-scale contact problems. This interest in not only driven by technological considerations, but also by inafordable CPU costs and breakdown of continuum contact mechanics at fine roughness scale (e.g. [3])

Our ultimate aim would be a contribution to bridging the gap between computational continuum contact mechanics and physics communities.This work is basically motivated by the fact that, though the physical models related to the fine contact scales are rather well understood and though very powerful computers do exist, the calculus of approximate numerical solutions of a resulting mono-model fine scale and engineering contact problem remains intractable. Hybrid methodologies are thus mandatory. Huge efforts of computational mechanics, physics and applied mathematics communities have been made to provide appropriate methodologies to address this issue.

This work is a contribution to this effort. Its ultimate goal is to provide a multi-physics and multi-numerics modeling for multiscale contact problems. We will be presenting a new partition of multi-level interface interactions models (paper in preparation) within the Bridging Domain H/Arlequin Method [5] (see also , [6, 7] for applications of the BDH/AM to multi-model contact problems). The first results derived by our doubly paritionned approach are promising. Both sides of the approach, all with the used global numerical strategy, will be explained during the conference. Numerical results for simple, still relevant 1D and 2D multi-physics and multiscale contact tests, introducing asperities, will also be shown and commented during the conference. An exemple is reported here: in Figure1, a two asperities toy contact test is shown. One can observe the

2 supper-imposed models within the H/Arlequin framework. In Figure2, the Lennard-Jones part of the contact interaction force is plotted and one can observe the attractive part of this force.

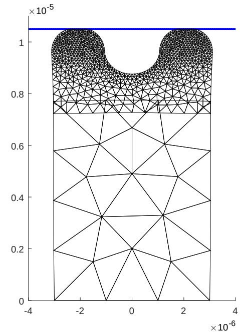

Figure 1: Two asperties contact test.

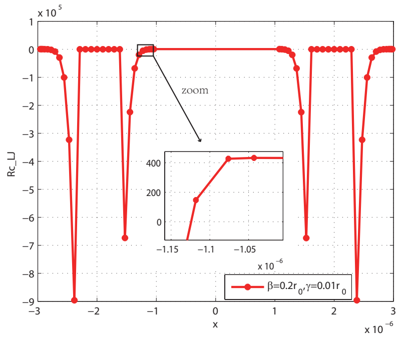

Figure 2: Lennard-Johns part of the surface interaction contact force.

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Contact Problems in Piezoelectricity - Mathematical Modelling and Boundary Element Approximation

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Summary: In this contribution we are concerned with unilateral contact problems in piezoelectricity. We consider various modelling cases of the boundary conditions: insulating foundation and frictionless contact, conducting foundation and friction. We apply potential methods to transform the weak formulation of these contact problems to boundary variational inequalities involving boundary integral operators. Then we study the numerical approximation of these contact problems and employ the boundary element method (BEM). Since we here admit approximations of higher order, we investigate nonconforming approximations.

Extended Abstract

In this contribution we are concerned with unilateral contact problems in piezoelectricity.

First we extend the mathematical modelling and solvability analysis given in [1] on transmission problems for piezoelectric elastic materials to unilateral frictionless contact with an insulating foundation and then turn to the more general frictional boundary conditions from [4, 5] that admit conducting contact with the foundation and nonmonotone frictional contact [5].

Similar to [3] for micropolar hemitropic elasticity, we use the fundamental solutions of linear elastostatics and electricity and apply potential methods which transform the weak formulation of these contact problems to boundary variational inequalities involving boundary integral operators. Based on our boundary variational inequality approach we prove existence and uniqueness theorems for weak solutions. We prove that the solutions continuously depend on the data of the original problem and on the friction coefficient. We treat also the case when the body is not fixed, but only submitted to some forces along some part of the boundary and is in unilateral frictional contact with a rigid foundation. In this situation we present necessary and sufficient conditions of solvability.

Then we study the numerical approximation of these contact problems. In virtue of our boundary variational inequality approach we can reduce the spatial dimension and employ finite element discretization on the boundary, only, what leads to the numerical treatment by the well-known boundary element method (BEM). Since we here admit approximations of higher order, we are confronted with nonconforming approximation. To this end we extend the numerical analysis given in [2] for a scalar model problem to the full vectorial case of coupled piezoelectricity.

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Thermo-mechanical contact between crack surfaces in the eXtended Finite Element Method

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Abstract

The eXtended Finite Element Method (XFEM) can be used for simulations of fracture mechanics problems. In XFEM cracks are considered at the element level and discontinuities in the displacement and temperature field are allowed. The method allows to determine accurately the crack opening displacement and crack tip stress near field.

Similar to multi body problems with contact, it is necessary to enforce the non-penetratio of crack surfaces, which in the XFEM occurs at the element level for the case of crack closing. Additionally, pressure depended heat conduction over crack surfaces should be taken into account for thermo-mechanically coupled problems which are investigated in this work. The penalty method is used to enforce the constraint equations for the displacements and to avoid unphysical penetration of the crack surfaces. The Node-to-Segment approach is chosen as contact detection method for the crack surfaces in a 10-Nodes tetrahedral elements. The contact formulation is based on the work by Mueller-Hoeppe et al.[1] and is implemented for normal contact. Results of mechanical/thermo-mechanical benchmark tests will be compared with the results achieved by the developed Finite element formulation.

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Contact algorithms for liquid droplets

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The theory and corresponding computational algorithms describing the contact behavior of liquid droplets is presented. In the hydrostatic case three different algorithms are required to capture the different contact states of the droplet. The formulation is illustrated by several numerical examples.

Introduction

Liquid droplets are governed by surface tension, and effectively behave like structural membranes enclosing a fluid volume. At the contact boundary they can form sharp contact angles that can vary along the contact line. Under hydrostatic conditions, droplets can be fully described by their surface geometry and the pressure of the enclosed liquid. In order to compute the shape of hydrostatic droplets, the surface membrane equations need to be stabilized. A finite element formulation for this was recently proposed in [1] based on the membrane formulation of [2].

Contact conditions for liquids

At the contact surface the usual impenetrability constraint

$$g_{\mathrm{n}} \geq 0 \tag{1}$$

is observed and can be enforced by the usual contact algorithms. Under hydrostatic conditions, tangential contact forces cannot be transferred across the contact surface, and so no frictional contact algorithms are needed for the contact surface. This can only change during droplet motion, when the fluid flow within the droplet imparts shear forces on the contact surface. But even under hydrostatic conditions, tangential contact forces can still be transferred at the contact line due to the contact angle. Two cases can be distinguished for the contact behavior along the contact line: Frictionless line contact, which occurs for a constant contact angle since the resultant tangential contact force is zero, and frictional line contact, which occurs for a varying contact angle since then a resultant tangential contact force occurs. For each case new kinds of contact algorithms are needed.

Algorithms for line contact

In the case of frictionless line contact, the contact conditions along the contact line can be enforced by simply applying a line load along the contact boundary. A corresponding algorithm for this was proposed in [1] based on the contact theory of [3]. In the case of frictional line contact, three states can be distinguished: contact line pinning (i.e. sticking), contact line advancing (i.e. forward sliding) and contact line receding (i.e. backward sliding). The three states are characterized by the sticking constraint

$$g_{\mathrm{t}}^{\alpha} = 0 \ , \quad \alpha = 1, 2, \tag{2}$$

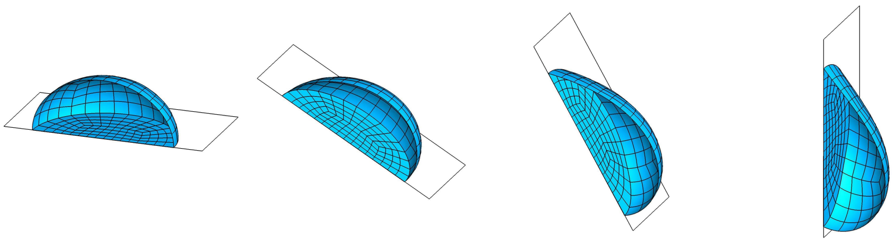
together with the contact angle range

$$\theta_\mathrm{r} \leq \theta_\mathrm{c} \leq \theta_\mathrm{a} \ , \tag{3}$$

where θ_a and θ_r are the limit values during advancing and receding. A predictor-corrector algorithm is formulated in order to determine wether pinning, advancing or receding occurs. It is presented in detail in a forthcoming publication.

Numerical example

Figure 1 shows a droplet under gravity loading on an inclined plane. The inclination of the plane is increased from $\beta = 0$ to $\beta = 90°$. The receding and advancing contact angles are chosen as $\theta_\mathrm{r} = 20°$ and $\theta_\mathrm{a} = 110°$. At about $\beta = 34°$, the contact angle at the leading edge reaches θ_a and starts advancing. At $\beta = 90°$ about a quarter of the contact line has advanced while the rest is still pinned.



(a) (b) (c) (d)

Figure 1: Liquid droplet on an inclined plane with inclination (a) $\beta = 0$, (b) $\beta = 30°$, (c) $\beta = 60°$ and (d) $\beta = 90°$. The leading edge of the droplet starts advancing at about $\beta = 34°$.

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Finite deformation effects in soft elastohydrodynamic lubrication problems

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Summary: Modelling o lubricated contacts operating in the soft elastohydrodynamic lubrication regime may require adequate treatment of finite deformations and the corresponding finite configuration changes. A finite-element formulation has been developed and implemented for steady-state problems and subsequently applied to study the related effects. An objective formulation of the Reynolds equation has also been derived for the case of time-dependent lubrication surface.

Soft elastohydrodynamic lubrication (soft-EHL) problems have recently attracted an increased research interest. This is due to numerous applications in technology (elastomeric seals, windscreen wipers, tyres, etc.), but also because this lubrication regime occurs in many biotribological systems (e.g., synovial joints, human skin contact, contact lenses, etc.). Contrary to the more common hard-EHL regime, the pressure is relatively low in soft-EHL contacts. Nevertheless, the elastohydrodynamic coupling is crucially important because one or both contacting bodies are compliant. Accordingly, relatively low contact pressures may lead to finite deformations of the contacting bodies. The corresponding effects have so far attracted little attention, and a study of those effects is pursued in this work.

Modelling of an EHL problem involves description of the fluid part, the solid part and the elastohydrodynamic coupling. The fluid part is conveniently modelled using the classical Reynolds equation. In the classical EHL theory, the solid part is usually modelled within the linear elasticity framework. Furthermore, the elasticity problem is usually formulated for a half-space for which specialized, highly efficient solution techniques are available. Both assumptions (linear elasticity and half-space approximation) are fully adequate for typical hard-EHL problems. However, the modelling framework must be enhanced once finite deformations and related configuration changes are to be considered.

A finite-element framework for simulation of *steady-state* finite-deformation soft-EHL problems has been developed in [1–3]. In this approach, the solid part is modelled using the finite element (FE) method so that an arbitrary (hyperelastic) material model and an arbitrary geometry can be adequately treated. The Reynolds equation is formulated on the contact boundary of the solid, and the solid-fluid coupling (lubricant film thickness depends on the deformation) and the fluid-solid coupling (the hydrodynamic pressure and the shear stress are applied to the body as the surface traction) are fully accounted for. The Reynolds equation is, in fact, formulated on an unknown *deformed* contact surface which introduces an additional coupling due to finite configuration changes. The problem is solved monolithically for all unknowns, i.e., displacements of the solid and lubricant pressures. The nonlinear FE equations are consistently linearized so that the Newton method can be efficiently used. The model employs a recently developed mixed formulation of the mass-conserving cavitation model [4], which is particularly suitable due to the specific mesh refinement technique used in the computational model.

The corresponding computational scheme has been implemented in the *AceGen/AceFEM* system [5], and the effects of finite configuration changes have been studied for two- and three-dimensional examples.

A more general case of a *time-dependent* lubrication surface has been recently studied by Temizer and Stupkiewicz [6]. Such a situation would be encountered in *non-stationary lubrication* conditions accompanied by finite deformations. It has been shown that the usual form of the Reynolds equation does not satisfy the objectivity requirement, and an objective formulation has been derived. An essential step in arriving at an objective formulation is an adequate definition of the relative velocity of two distant surfaces. This feature is well recognized in the contact mechanics community [7]. The new formulation of the Reynolds equations consistently describes finite-deformation effects related to curvature and surface expansion, as illustrated by analytical examples.

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